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## THE MARSHALLIAN DEMAND CURVE<sup>1</sup>

MARTIN J. BAILEY

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IN AN article with the above title, Professor Friedman<sup>2</sup> has urged that a constant-real-income demand curve is a more satisfactory tool for economic analysis than the customary constant-other-prices-and-money-incomes demand curve and that, at least in the first two editions of the *Principles*, this was the type of demand curve which Marshall really had in mind. On the latter, historical question nothing will be said here; but on the former, analytical question I shall contend that Friedman did not make the best choice of a curve as an improvement on the conventional one and that the constant-real-income curve, strictly interpreted, does not on balance possess the superiority he claims for it. Of the various interesting alternative types of demand curve which can be defined, one at least possesses most, if not all, of the advantages which Friedman can claim for any type of constant-real-income demand curve and none of its disadvantages.

In his argument in support of the constant-real-income demand curve Friedman demonstrated that the use of an ordinary demand curve in a demand-supply diagram to show the effects of a subsidy on a given commodity fails to take account of the necessary withdrawal of resources from oth-

<sup>1</sup> I wish to thank Mr. Amotz Morag and Professors Arnold C. Harberger and Carl F. Christ for their helpful advice and criticisms of early drafts of this note; and I wish to thank Professor Milton Friedman for his advice and criticism at a later stage. Specific acknowledgments to Professor Friedman appear at appropriate points in the text of this note. Responsibility for such errors as remain is, of course, my own.

<sup>2</sup> Milton Friedman, "The Marshallian Demand Curve," *Journal of Political Economy*, LVII (1949), 463-95.

er uses; on the other hand, the constant-real-income demand curve, which in the limit is an approximation of what the community can actually have, allows for this withdrawal of resources and therefore presents a better picture of the final outcome.<sup>3</sup> While Friedman's analysis does not contain any errors, it is liable to serious misinterpretation if its assumptions and their relevance are overlooked; on the other hand, with a different type of demand curve which I shall propose the pitfalls can be avoided, and an analytically superior tool can be had in the bargain.

### DEMAND CURVES AND PRODUCTION POSSIBILITIES

Suppose, for simplicity of arrangement, that a fully employed community has the production possibilities between its two competitively produced commodities  $X$  and  $Y$  as shown by the opportunity-cost curve  $ST$  in Figure 1A. Money, different from either commodity, is used as a unit of account only; money incomes are assumed to be spent in full, and the absolute price level to be determined arbitrarily.<sup>4</sup>

From the community indifference curves (for the moment assumed to be defined unambiguously) shown in Figure 1A, we may derive the two demand curves mentioned so far (the constant-real-income and the other-things-equal demand curves) in the customary manner.  $DD$  in Figure 1B is defined by the price-consumption line  $PC$  in Figure 1A, and  $RR$  is obtained from the equilibrium

<sup>3</sup> *Ibid.*, pp. 467-74.

<sup>4</sup> Friedman's assumption of a fixed supply of factor services is retained here, since its retention does not cause any loss of generality in the argument.

indifference curve  $I_1$  by noting the quantity of  $X$  at which  $I_1$  has any given slope (i.e., marginal rate of substitution, interpreted as a price ratio,  $P_x/P_y$ ).

Suppose now that the government pays a subsidy on production of  $X$ ; the apparent effect after production adjusts itself to the new conditions will be to lower the price of  $X$  by some fraction of the amount of the

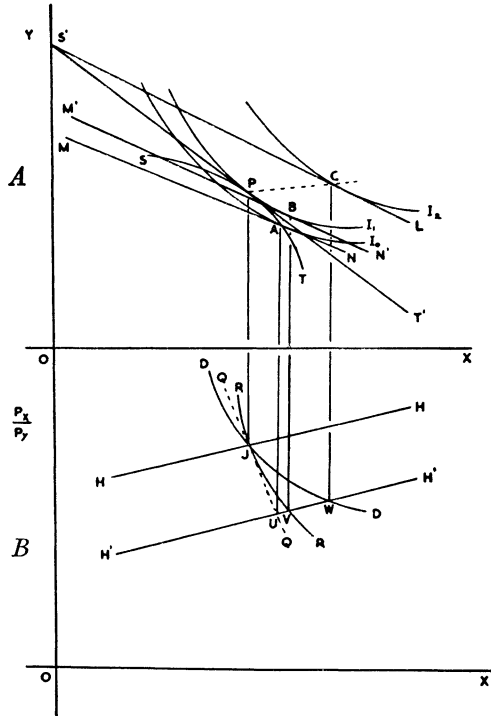


FIG. 1

subsidy, changing the price line from  $S'T'$  to  $S'L$  in Figure 1A, and to leave the price of  $Y$  and money income unchanged. Given this apparent opportunity, the community would like to consume to the point  $C$  in Figure 1A, that is, to the point  $W$  in Figure 1B. However, as Friedman pointed out, this is clearly impossible. Physical supplies are not available, and corresponding to this lack there is an inflationary gap equal to the going amount of the subsidy; also the relative price of  $Y$  must fall owing to the shift of production toward more  $X$ .

Hence we must further suppose that the

government imposes an income tax always equal to the subsidy. The final equilibrium point is found where a price line which is tangent to an indifference curve where it crosses the production frontier differs in slope from the slope of the production frontier at that point by an amount corresponding to the subsidy.  $S'L$  will "shift" to  $MN$ , where it is tangent to the indifference curve  $I_0$ , lower than  $I_1$ , at  $A$ . This equilibrium point is only slightly distant from  $B$ , the point at which  $M'N'$  is tangent to  $I_1$ . ( $S'L$ ,  $M'N'$ , and  $MN$  have in common that each one's slope differs by the rate of subsidy from the slope of the production frontier beneath the point where each one is tangent to an indifference curve.)

It can be seen from this result that neither  $DD$  nor  $RR$ , in Figure 1B, shows the final outcome correctly. The correct outcome could be obtained only from another type of demand curve, the "production-frontier" demand curve, which would show, for each amount of  $X$ , the marginal rate of substitution of the indifference curve which crosses the production frontier at the point where that amount of  $X$  is produced. This demand curve is shown as  $QQ$  in Figure 1B; if  $X$  is not an inferior good,  $QQ$  must lie to the left of both  $RR$  and  $DD$  below  $J$  and must lie between them above  $J$  (where  $J$  in Fig. 1B corresponds to  $P$  in Fig. 1A). Its intersection with  $H'H'$  at  $U$ , corresponding to  $A$  in Figure 1A, shows the true outcome as the result of the imposition of the combination subsidy and income tax.

The production-frontier demand curve is clearly the one hypothetically most desirable for use in the comparative statics of demand analysis, since it shows what in fact the community will take when the repercussions on the production of other commodities are taken into account. Its weakness is that it is defined only for given production conditions. Presumably tastes are relatively constant, whereas real or apparent production conditions are always changing because of fluctuations in weather and crops, changes in government policy, and other factors. Data on market behavior may, to

the extent that this is true, be supposed to tell us something about consumer preferences but to tell us little about production conditions. At any moment of time, however, production conditions are in some sense fixed; and for economic analysis it would be desirable to take these conditions into account in analyzing demand. Lacking knowledge of these conditions of the moment, we must adopt some more or less arbitrary method of approximating the effects of a change in policy or the like.

Friedman argues in effect that  $RR$  (in Fig. 1B) is a better approximation to  $QQ$  than is  $DD$ , since  $I_1$  is tangent to  $ST$  at  $P$  and so approximates it in the limit, whereas  $PC$  has no such limiting property. That is to say,  $RR$  is tangent to  $QQ$ , but  $DD$  is not. This is correct, as long as the community preference field (the function represented by the indifference map) is innocent of any discontinuities in the first and all higher derivatives. Though I suppose there is no reason to doubt its innocence for practical purposes, this qualification should be recognized as relevant. But other arguments developed below substantially weaken the case for the constant-real-income demand curve.

#### THE CONSTANT-REAL-INCOME CONCEPT

The argument so far has been greatly aided by the use of unexplained community indifference curves. It is now necessary to investigate the meaning of these curves of constant community real income and of the idea of a constant-real-income demand curve. The construction of community indifference curves will not be repeated here; suffice it to say that constant community real income means constant real income for every individual in the community.<sup>5</sup> The relevant construction necessarily implies the existence of different distributions of money incomes at different points along a given

<sup>5</sup> William J. Baumol, "The Community Indifference Map: A Construction," *Review of Economic Studies*, XVII (1949-50), 189-97; and E. J. Mishan, "The Principle of Compensation Reconsidered," *Journal of Political Economy*, LX (1952), 514-17.

community indifference curve; the reason for this will become clear in the following discussion.

Consider, in Figure 2, the indifference curves of two individuals whose money incomes are equal.<sup>6</sup> When the two indifference maps are superimposed on one another, their opportunity lines will coincide, as, for example, in  $AB$ . The individual  $I$  will be in equilibrium at  $P$ , and the individual  $J$  will be in equilibrium at  $Q$ , given the oppor-

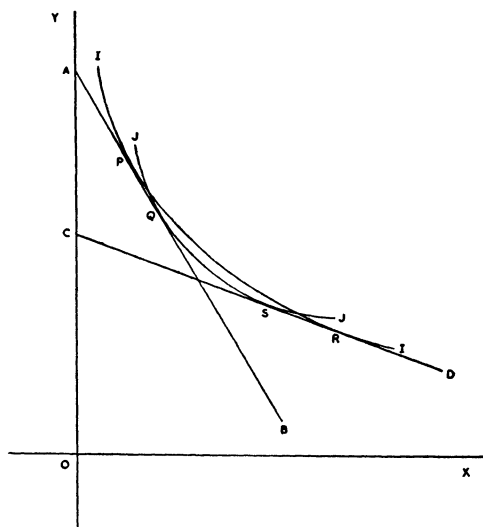


FIG. 2

tunity line  $AB$ . Now for an arbitrary change in the price of, say, commodity  $Y$ , what price change of  $X$  will keep both individuals on the same levels of real income  $J$  and  $I$ ? It is at once apparent that there need not be any price change of  $X$  which will do the trick. If the price of  $Y$  should rise until the given money income of each individual could purchase only  $OC$  of  $Y$ , then a price of  $X$  corresponding to the opportunity line  $CD$  would do it, since  $CD$  happens to be

<sup>6</sup> For persons with different money incomes, the scales of  $X$  and  $Y$  quantities for the person with the larger income may be compressed (in the same proportion) until the two opportunity lines coincide when the indifference maps are superimposed. The argument in the text then applies without change to this case.

tangent to both  $I$  and  $J$  at  $R$  and  $S$ , respectively. But the set of points  $C$  through which a line can be drawn tangent to both indifference curves is in general a finite set (the principal exception being the case where the two indifference curves coincide) and may be empty, aside from the point  $A$ . A price compensating constant-real-income demand function for the two individuals must remain undefined except at points such as  $C$ —that is, we cannot, in general, have a constant-real-income demand “curve” at all, as long as money incomes are held constant.

On the other hand, if money income changes are used—in general, a different change for each individual—then it will always be possible to find an income change for each individual that will just offset any price change (or set of price changes) and permit him to achieve the same indifference curve as before. This, in effect, is what is done in defining community indifference curves.

But if the method of compensating price changes is used, there is no such thing as a constant-real-income demand curve for two individuals taken together. Such a curve can be defined for each one, but the curves cannot be aggregated because the price changes of  $Y$  offsetting a given price change of  $X$  would be different for the two individuals. This would be true a fortiori for a larger community; and it would continue to be true whatever the number of commodities.

It should be clear, then, that a constant-real-income demand curve for a community cannot be defined in terms solely of offsetting price movements for all possible price changes of a given commodity unless everybody's tastes are, in effect, identical. In fact, identity of tastes is not sufficient when money incomes are different. What is required is that the indifference curve on which each individual finds himself in equilibrium must be an exact projection of the corresponding indifference curve of every other individual. Unless all indifference systems were homogeneous, identity of

tastes would guarantee this coincidence only for an equal distribution of income.

TWO APPROXIMATIONS: CONSTANT APPARENT REAL INCOME AND CONSTANT APPARENT PRODUCTION

The objections against a constant-real-income demand curve, as I have so far defined it, are for any practical purpose overwhelming; recourse may be had, however, to an approximating concept which avoids these objections.<sup>7</sup> This concept is that of the constant-*apparent*-real-income demand curve, which can be defined for constant-money incomes all around and with no particular knowledge of individual consumer preferences. In Figure 3A the point  $P$  represents, as before, the initial equilibrium point, and  $S'T'$  is the equilibrium price line. If the price of  $X$  is lowered, the consumers' real income will “apparently” be the same if the price of  $Y$  is raised to the point where the consumers are just able to buy the same bill of goods they bought before; that is, the new price line  $M''N''$  should pass through  $P$ . This, to a first order of approximation, cancels out the income effect to consumers in the aggregate<sup>8</sup> but allows them a small gain

<sup>7</sup> In his text Friedman uses the constant-apparent-real-income demand curve (*op. cit.*, pp. 466–67).

<sup>8</sup> If individual incomes are not adjusted, then “income effects” are not removed by this procedure even to a first order of approximation for individuals, since no individual need be consuming the two commodities in the same proportions as they are consumed by the whole community.

However, this consideration may be ignored for the constant-apparent-real-income demand curve, if we like, whereas in the nature of the case it cannot be ignored for the “true” constant-real-income demand curve. Furthermore, if we choose not to ignore it, we need only to know the original quantities bought by each consumer in order to define the constant-apparent-real-income demand curve, whereas for the constant-real-income demand curve one must know the shape and position of each consumer's relevant indifference curve. Similar remarks apply to the constant-apparent-production demand curve discussed below in the text.

So far as I can see, the production-frontier demand curve has the disadvantage that there is no logical way to define it for each individual in the community—it is a purely aggregate function, and any relative income distribution is consistent with

in "real income" by substituting  $X$  for  $Y$ ; the new bill of goods they would choose if they had this opportunity would be  $B'$ , on the community indifference curve  $I_3$ , higher than  $I_1$ .

The demand curve derived in this way is not the same thing as the true constant-real-income demand curve as previously defined (which depended on the shape of  $I_1$  only), but it can be proved to be a first-order approximation of it,<sup>9</sup> just as the true constant-real-income demand curve is a first-order approximation of the production-frontier demand curve. It follows that the constant-apparent-real-income demand curve is a first-order approximation of the production-frontier demand curve. Furthermore, it does not suffer from the difficulties of definition of the other curve, since it can unambiguously be defined in terms of constant-money incomes for every individual.

In practice, something in the nature of a constant-apparent-real-income demand curve could be derived statistically from ordinary total market data; whereas a true constant-real-income demand curve could not but would require data on every individual. With a statistically derived demand curve in our hands, we would not know what values of the price variables (if any) would give every consumer the same real income (for a constant-money income) as some other set of values of the price variables. However, it would be a simple matter to choose a set of price variables giving the same *apparent* real income (as here defined) to the community as some other set; all that has to be done is to choose a set of prices

its definition. This disadvantage is the antithesis of the disadvantage of the constant-real-income demand curve, which in effect is defined only for the individual.

My earlier omission of the points in this footnote was brought to my attention by Friedman.

<sup>9</sup> See Jacob L. Mosak, "On the interpretation of the Fundamental Equation of Value Theory," in O. Lange *et al.* (eds.), *Studies in Mathematical Economics and Econometrics* (Chicago: University of Chicago Press, 1942), p. 73 n.

which keeps a base-weights price index unchanged in value.<sup>10</sup>

However, the possibilities for better practical approximation of the production-frontier demand curve are not yet exhausted. We may with comparable simplicity define a constant-apparent-*production* demand curve; and this will be the best approxima-

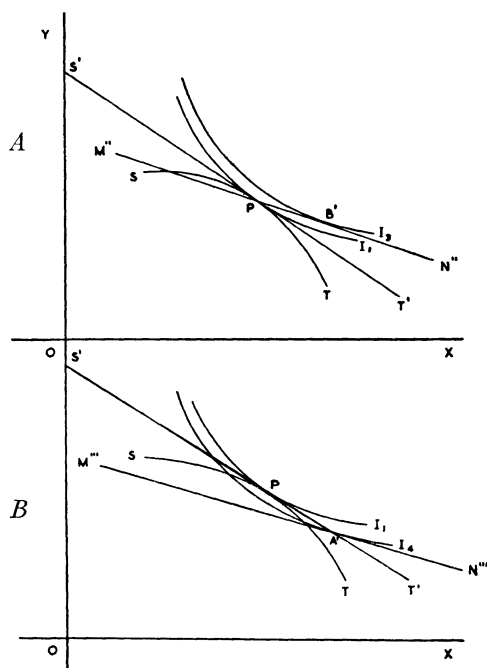


FIG. 3

tion of the lot. In Figure 3B the line  $S'T'$  represents the equilibrium price line as before, and, being tangent to the production frontier  $ST$  at  $P$ , it represents a local approximation of  $ST$ , just as does  $I_1$ . A useful demand concept is defined by moving along  $S'T'$ : for any given price ratio for  $X$  and  $Y$ , we obtain from the community indifference map that bill of goods among those along  $S'T'$  which the community would prefer; that is, we find the community indifference curve  $I_4$ , which at its point of crossing of  $S'T'$  has the same slope as the given price line,  $M'''N'''$ .

We may now compare the different con-

<sup>10</sup> Friedman, *op. cit.*, p. 467.

ceptions of demand set forth here; the curves are illustrated in Figure 4, which is derived from Figures 1A, 3A, and 3B in the same manner as Figure 1B is derived from Figure 1A. The curves  $DD$ ,  $RR$ , and  $QQ$  in Figure 4 are the same as in Figure 1A; the new curves  $R'R'$  and  $Q'Q'$  are the approximations—constant *apparent* real income and production, respectively—discussed in this section.

The curves  $R'R'$ ,  $RR$ , and  $Q'Q'$  are all tangent to  $QQ$  at  $J$ , a condition which will hold provided the necessary continuity ob-

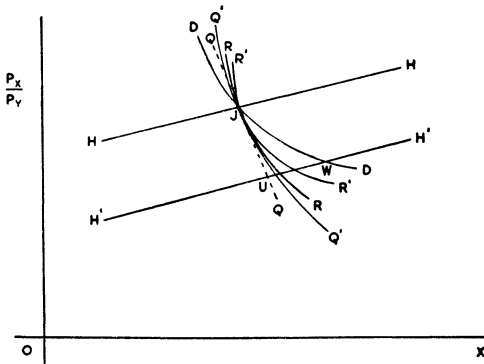


FIG. 4

tains in preference and production; and it can also be seen that  $R'R'$ ,  $RR$ , and  $Q'Q'$  are successively better approximations in that order to  $QQ$ , which represents the demand derived from what the community can actually have. (The relative positions of the various curves depend on the assumption that commodity  $X$  is not inferior.) No importance should be attached to the absolute curvatures of the different curves, which depend on the conditions of preference and production; but under the assumed conditions it is necessarily true that  $QQ$ ,  $Q'Q'$ ,  $RR$ , and  $R'R'$  are successively more concave upward and that  $Q'Q'$  is the best approximation to the shape of  $QQ$ .

The constant-apparent-production demand curve can, like the constant-apparent-real-income demand curve, be derived from market data on quantities sold and prices.

Just as the constant-apparent-real-income demand curve is obtained from the knowledge of the original equilibrium quantities and of the relevant part of the consumer preference field as revealed in market data, so the constant-apparent-production curve is obtained from a knowledge of the original equilibrium *prices* and of the relevant part of the consumer preference field. The first involves keeping a base-weights price index constant; the second involves keeping a base-weights quantity index constant. Such awkwardness of definition as exists in the constant-apparent-production demand curve disappears if the Continental procedure of expressing prices as a function of quantity is adopted.<sup>11</sup>

The constant-apparent-production curve has the advantage, however, that it represents the true possibilities closer than does the constant-real-income demand curve. *It utilizes information which the latter curve does not use, namely, that the equilibrium price ratio is itself an approximation of the alternative bills of goods which the community can in fact have.*

There is one other point on which the suggested "improvement" of the conventional demand curve might be rejected: the conventional demand curve is unambiguous about how "other prices" behave, whereas none of the other demand curves is. If there are several commodities, a given change in the price of commodity  $X$  may be offset by price changes in other goods in any of a number of different ways still meeting the specifications of the other four types of demand curves. It may make a good deal of difference to the demand for  $X$  whether the prices of closely competing or complementary goods are changed a little or a lot to compensate for the change in the price of  $X$ . If any demand curve other than the all-other-prices-and-incomes-equal demand curve is used, some arbitrary specification must be made as to how other prices are to change to offset changes in the price of  $X$ , such as that all other prices change in the

<sup>11</sup> I am indebted to Friedman for this point.

same proportion. It should be recognized that such a solution is arbitrary, since whatever choice is made does not necessarily have any connection with the way these prices would really change if, say, a subsidy were imposed on commodity *X*. The conventional demand curve solves this problem (also arbitrarily, of course) by assuming that other prices do not change at all.

#### FINAL COMMENT

The conclusion of the above remarks finds me substantially in agreement with Friedman's argument in favor of revising the conventional notion of a demand curve when we desire to analyze the effects of an excise tax or subsidy, although I have come out in favor of even greater revision than he suggested. In the policy problem in question, the community's production opportunities are unaffected, but *apparent* supply conditions are changed. Therefore, it is simplest to use a demand curve along which true supply conditions are (exactly or approximately) unchanged. The conventional demand curve does not meet this specification; consequently, in the problem under consideration one must show a shift in such a demand curve, as well as in the apparent supply curve, as the effect of the policy action.<sup>12</sup> If market data are sufficiently informative, both demand and supply conditions are hypothetically ascertainable, and the production-frontier demand curve may be used. If not, the approximations discussed here may be used, the better of which

<sup>12</sup> Friedman, *op. cit.*

is the constant-apparent-production demand curve.

The situation is not the same if the problem under consideration involves changes in actual supply conditions such as (a) changes in technique, (b) crop variations and the like, and (c) changes in government activity, altering the availabilities to the private sector of the economy. In any such case the relevant demand curve would change, that is, would "shift." This is true of the production-frontier demand curve and of all three of its approximations. It is possible that, by coincidence, the new equilibrium might be on the old price-consumption line (*PC* in Fig. 1A); and in this case the conventional demand curve would give the true result without shifting. No such coincidence is possible for the other four demand curves if the new production frontier lies entirely above or below the old one. Beyond this, however, nothing can be said as to whether the outcome of a change in conditions can or cannot be approximated with any single demand curve defined here.

It is therefore evident that the choice of a demand curve for purposes of analysis should depend on the problem in hand; and for some problems no demand curve will perform with the simplicity we might desire. It should therefore also be evident that the use of general equilibrium diagrams such as Figure 1A is an important supplement to clear and accurate analysis. With such diagrams it is still necessary to state the relevant qualifications regarding income distribution, but subject to this the interrelationships between different types of changes in conditions can be shown.

#### A REPLY

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**B**OTH my own earlier discussion of the definition of the demand curve and Bailey's interesting comments are phrased in terms of the variables that it is appropriate to hold constant along a demand curve

viewed as a two-dimensional relation between quantity and price. It may help in bringing out the issues involved in this choice to present the problem in rather different terms.