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The Geometry of Capital and Interest: A Suggested Simplification

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REFERENCES

1. J. G. CRAWFORD, "Tariff Level Indices," *Econ. Record*, Dec. 1934, 10, 213-21.
2. G. HABERLER, *The Theory of International Trade*. London 1937.
3. A. LOVEDAY, "The Measurement of Tariff Levels," *Jour. Royal Stat. Soc.*, 1929, 92, Pt. 4, 487-516.
4. *The Australian Tariff: An Economic Enquiry*. Melbourne 1929.
5. *Customs and Excise Tariffs* (2nd Revision T.D.58/1066). Canberra, Australia.
6. *League of Nations, Report on Tariff Indices 1927* (Doc. C.E.I.37). Geneva 1927.
7. U.S. CONGRESS, Subcommittee on Foreign Economic Policy of the Joint Economic Committee, *Trade Restraints in the Western Community*, 87th Cong., 1st sess., Washington 1961.

The Geometry of Capital and Interest: a Suggested Simplification

In recent years, the study of capital and interest has come to be viewed as a branch of monetary theory. No doubt interest is, in some degree, a monetary phenomenon. (If it is not, central bank policy is an awful hoax.) Unfortunately, young economists who first make the acquaintance of capital and interest through their work in monetary theory often seem to believe that interest is mainly a monetary phenomenon. They are, of course, badly informed.

Since Böhm-Bawerk's day the well-informed economist has known that interest is rooted mainly in the productivity of investment and the economy's preference for present over future consumption. The effective exposition of this truth, however, has never been easy, and for the last thirty years pedagogical progress has been distressingly slow. Monetary theorists now have a large bag of elegant tricks that demonstrate how changes in the money supply and liquidity preference can affect the interest rate. Advocates of the so-called "real" theory of interest must fall back on a set of teaching aids with a distinctly musty odor—Robinson Crusoe and his fish-catching activities, growing trees, aging wine, the more roundabout method of production, etc. Therefore, there may be merit in describing a pedagogical device that allows the essential properties of capital and interest to be simply, albeit rigorously, conveyed.

Two of the more effective tools available are Irving Fisher's indifference curves [2, pp. 387-90] and Frank Knight's Crusonia plant—a species of vegetation that supplies all human wants and, though unattended, grows at a constant (geometric) rate "except as new tissue is cut away for consumption" [3, p. 30]. In essence, the pedagogical device described here applies, with modifications, Fisher's geometry to Knight's Crusonia plant.¹

¹The Crusonia plant has the great virtue of eliminating three complications in capital theory that are fundamentally unimportant but are, nonetheless, time-consuming in the classroom and a source of confusion to the young. It enables us to ignore the "period of production"—the gestation period of capital assets. (For the view that the period of production is an important idea, see Robert Dorfman's rigorous restatement of Böhm-Bawerk's theory [1].) This period is perforce zero since the Crusonia plant grows con-

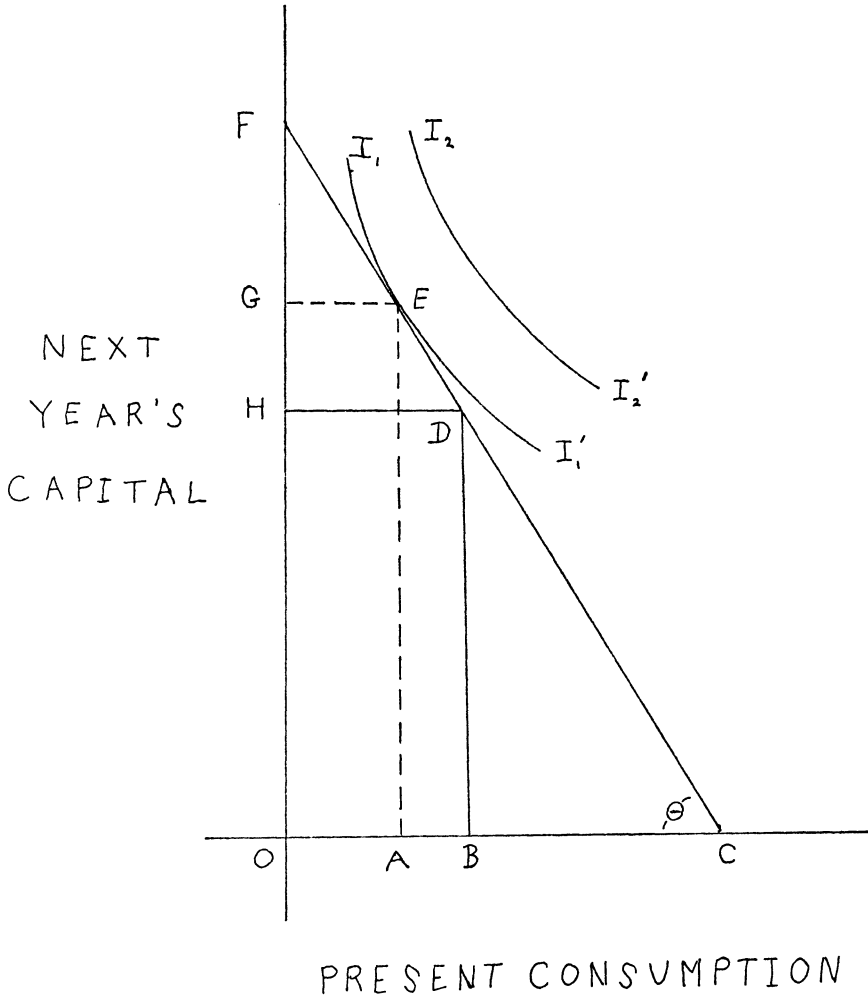


FIGURE 1

Consider Figure 1. Measure present consumption on the horizontal axis OC . We posit that all consumption for the coming year is done "now." This assumption allows us to treat consumption as having the dimensions of a stock rather than of a flow and, hence, to represent consumption and the stock of capital on the same axis. In Figure 1, OC denotes the maximum amount of consumption that is possible now. OC also denotes the present stock of capital

tinuously. It allows us to avoid all measurement problems. The stock of capital, consumption, and investment per unit of time, can be expressed as quantities of Crusonia. And finally, the Crusonia plant permits an exposition of capital theory in the first instance without any attention to the mechanics of capitalization and discount.

(Crusonia) on the assumption that no consumption for this year has yet occurred, and that the whole of the present capital stock can, if it is desired, be consumed this year.

Now measure the stock of capital that will be in existence (and available for consumption) one year from now on the vertical axis OF . If no consumption takes place now, the stock of capital will reach OF by the end of the year. Clearly the size of this future capital stock is a function of (a) the marginal productivity of investment, i.e., of not consuming, and (b) the amount consumed now. Every possible combination of present consumption and next year's capital stock is represented by a point on the line CF .

On OF mark off the distance OH equal to OC . The marginal productivity of investment (mpi) is given by the slope of the line CF (minus one). Since this slope is a constant, there are no diminishing returns to investment. The rate of growth of the capital stock is independent of its size, CF is perforce a straight line, and mpi equals $\tan \theta - 1$. We perceive from Figure 1 that OB is the amount of consumption compatible with a capital stock that reaches a maximum of OC (or OH) each year. Immediately after consumption OB , the present capital stock is BC . During the year it grows back to OC (or OH). Should consumption exceed OB , the capital stock will be smaller one year from now. Should some part of OB be invested, it will be larger.

In order to reckon investment—and hence capital growth—we must specify the economy's preference for present consumption over future capital. This we do in Figure 1 by a set of Fisher-type indifference curves, I_1I_1' , I_2I_2' . Present consumption is again measured along the horizontal axis and the stock of capital one year from now along the vertical axis.²

From Figure 1 we see that the economy attains its highest indifference curve (I_1I_1') by consuming OA now and having a capital stock of OG in one year's time. We may also read off the following information. For a period of one year:

- $OC = OH =$ initial capital stock.
- $OG =$ final capital stock.
- $OB =$ income available for consumption.
- $OA =$ actual consumption.
- $AB =$ income invested.
- $HG =$ growth in capital stock ("net investment").
- $\frac{HG}{OH} =$ capital stock growth rate.

²In Fisher's original geometry the vertical axis measures future income and the horizontal axis present income [2, p. 387]. In that context, however, Fisher is not using "income" to denote a flow of goods (or services) in perpetuity. The value of the capital stock in his diagram can only be obtained by discounting the future income by the interest rate and adding the result to present income. Thus it is not immediately clear from Fisher's geometry whether a failure to consume all of present income (as defined by Fisher) serves to increase the capital stock or merely retard its decline. This uncertainty can be resolved by using the data provided by Fisher in his diagram; but the computation is tedious.

In Figure 1 the marginal productivity of investment during the year is given by any of the following:

$$\tan \theta - 1, \frac{OF}{OC} - 1, \frac{HG}{AB} - 1, \frac{HF}{OB} - 1, \frac{HF}{OH}.$$

Figure 1 depicts the "normal" case of an economy that is increasing its capital stock. To derive a zero growth rate we need only revise our diagram to make the indifference curve, I_1I_1' tangent to the line CF at point D . To show capital consumption we would make the point of tangency lie between D and C .

The geometry of Figure 1 has an additional merit as a teaching aid. It can be used to show that there is no genuine conflict between writers who treat capital as a stock of "real" things that yield incomes in perpetuity and those who define it as a sum of future incomes discounted by the rate of interest. (Given perfect arbitrage, the rate of interest must, of course, equal the marginal productivity of investment.) For example, let the perpetual annual income be HF ; let HF be paid in a lump sum once a year; and let the first payment be due one year from today. The present value of a perpetuity disbursed in this manner is equal to the annual income divided by the interest rate.

The interest rate is $\frac{HF}{OH}$. Therefore, $\frac{HF}{\frac{HF}{OH}} = OH$.

But from Figure 1 it is immediately apparent that, when consumption takes place on the last day of the year rather than on the first (as we previously assumed), HF is the annual harvest or income of the capital stock OC or OH .

Our geometry is easily modified to accommodate the case where the marginal productivity of investment declines as the capital stock increases. This is done in Figure 2. Let us revert to the assumption that all consumption for the year takes place now. Once again OC denotes the present stock of capital, OB the quantity of present consumption compatible with a constant stock of capital OC (or OH), and the arc FC every possible combination of present consumption and future capital.

However, when diminishing returns to investment are introduced into our model, a complication appears. There is now a different mpi for every different capital stock. And, since in the absence of consumption, the capital stock grows continuously, there is a different mpi for every moment during the year. (Recall that mpi is what the instantaneous growth rate of the capital stock would be if consumption were zero.) Thus one is reduced to speaking of the average value of mpi for the year. The greater the fraction of income invested now, the smaller the average value of mpi .

Figure 2 indicates that the economy has achieved long-run equilibrium. It attains its highest indifference curve II' by consuming the whole of its annual income OB . Average mpi is equal to the slope of FC and II' at point D (minus one). Note that when diminishing returns to investment are posited, average

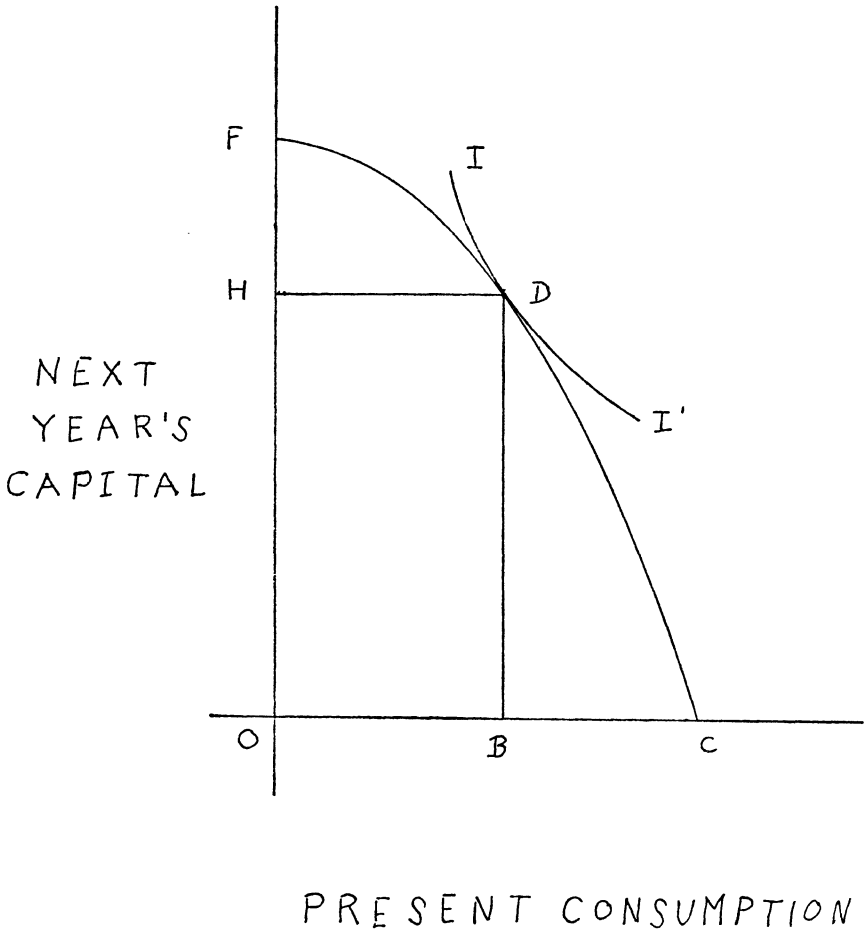


FIGURE 2

mpi is necessarily less than the average rate of return on investment during the year.

In Figure 2 the quantity of consumption invested now is BC , and the capital stock one year hence will be OH (or BD). The average rate of return on investment BC is therefore $OH/BC - 1$. This last rate has no special significance in capital theory, except that it is sometimes mistaken for the annual market rate of interest. But the interest rate is tied by arbitrage to average mpi —to the slope of FC at point D (minus one) in the present example. Indeed, the equality of average mpi and the interest rate is implied by the proposition that the economy has achieved its highest indifference curve.

What is true is that, as the long-run equilibrium of a stable capital stock

is approached, the interest rate and average *mpi* tend toward the average rate of return on *income* invested. For now the quantity of income invested becomes an ever smaller percentage of total investment, i.e., the capital stock. Thus in Figure 2, if some minute portion of income *OB* were saved, the average rate of return on income invested would be virtually the same as average *mpi*. In effect, the small fraction of *OB* saved would be the incremental unit of investment.

The essential properties of capital, income, consumption, investment, capital growth, time preference, and capital productivity can be set forth in many ways. The method described above has, I believe, the merit of allowing the delineation to be made clearly, compactly, and with minimum use of cumbersome notation.

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REFERENCES

1. R. DORFMAN, "Waiting and the Period of Production," *Quart. Jour. Econ.*, Aug. 1959, 73, 351-72.
2. I. FISHER, *The Rate of Interest*. New York 1907.
3. F. H. KNIGHT, "Diminishing Returns from Investment," *Jour. Pol. Econ.*, Mar. 1944, 52, 26-47.

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The Criterion for an Adverse Balance of Payments

I should like to register a protest with regard to the criteria now being used by the Department of Commerce with respect to whether changes in the balance of payments are adverse.

The following table consists of Lines 30 through 48 extracted from the Second Quarter 1962 data for the Balance of Payments of the United States, as presented in the September 1962 *Survey of Current Business* (pp. 14-15; in millions of dollars):

30	U.S. capital, net [increase in U.S. assets (-)]	-1,101
31	Private, net	-686
32	Direct investments, net	-449
33	New issues of foreign securities	-319
34	Redemptions	21
35	Transactions in outstanding foreign securities	-17
36	Other long-term, net	-80
37	Short-term, net	158
38	Government, net	-415
39	Long-term capital	-511
40	Repayments	212
41	Foreign currency holdings and short-term claims, net [increase (-)]	-116
42	Foreign capital, net [increase in U.S. liabilities (+)]	609
43	Direct investments in the United States	49