CHAPTER THREE

Competitive Exchange

The Assumptions of the Perfect Market

THUS FAR we have been discussing the reactions and relationships of consumers, individually or in groups, to one or more given market situations. We come now to examine how the interactions of the various groups in the market determine the market situation.

To keep the analysis tractably simple, we begin with the concept of the perfect frictionless market, in which the basic additional assumption needed, in addition to the general assumptions of the static economy, is that the process of exchange is without cost (or at least that these costs are negligible in magnitude). This in effect means that the price, or the slope of the opportunity path, with which a buyer is confronted is the same as that with which the seller is confronted; i.e., if the buyer of meat gives up three yards of cloth for ten pounds of meat, the seller will receive three yards of cloth for each ten pounds of meat that he surrenders. If there were costs of exchange, as in practice there always are, the seller would perhaps have to give up eleven pounds of meat, or the equivalent, for each three yards of cloth, one pound being lost in some way in the process of exchange.

Not only is the actual physical exchange supposed to be costless, but the process of bringing together buyers and sellers is likewise assumed to be costless, so that in effect there is no obstacle to any mutually advantageous exchange taking place. Each trader is supposed to have complete information concerning all transactions in which he might become interested, or at least information as to the prices at which such transactions take place. This leads to there being only one price in the market at any given time for any particular commodity; two separate transactions between pairs of buyers and sellers undertaken at separate prices, each pair aware of the other transaction, would presumably lead to action by the seller getting the lower price to try to get some of the advantage of the higher price, and conversely for the buyer paying the higher price, which would tend to result in the prices being brought into line with each other.

In a general theory of exchange, it is supposed that the several commodities can be exchanged directly for one another (i.e., by direct barter or exchange without the necessary intervention of money or any other "medium of

exchange"). Thus there may be exchange rates between each pair of commodities. However, in a frictionless market the various prices must be in such a relation that the amount of one commodity secured in exchange for another must be the same whether the exchange be direct or whether it be carried out in two or more stages through the purchase and resale of intermediate commodities. If this were not so there would be opportunities for profit through "arbitrage," i.e., for a series of exchanges to be carried out in a complete circuit in such a way that the trader would come out with more than he started with of some commodity and at least as much of all others; the effect of such transactions would be to reduce the discrepancies until they vanished. An actual example of this is found in the operation of international exchange in the absence of artificial restrictions: if francs are selling 400 to the dollar, and if dollars are selling three to the pound, then necessarily the exchange rate between francs and sterling must be very closely 1,200 to the pound, plus or minus a small margin representing brokerage costs, for if substantially more than 1,200 francs could be had for a pound, the owner of francs could buy dollars, with the dollars secured buy pounds, and with the pounds so secured buy more francs than he originally sold for dollars; conversely, if a pound is obtainable for substantially less than 1,200 francs, the reverse sequence of exchanges would yield a profit.

Since this relationship must exist between the various exchange ratios, all exchange ratios will be determined once the price of each commodity in terms of any single commodity is given. Accordingly, in order to reduce the amount of confusion in talking about prices, it is customary to select some commodity arbitrarily as a numéraire, in terms of which the prices of all the other commodities are expressed. This choice of a numéraire is purely arbitrary, and has nothing to do with the role of the selected commodity in actual exchange; in fact, there is nothing to prevent the selection of a commodity as numéraire which itself enters only relatively infrequently into exchanges, and perhaps even which is exchanged directly with only one other commodity. Two different investigators might describe a given situation in terms of completely different numéraires, and still come to exactly the same concrete conclusions. The concept of a numéraire should not be confused with other functions of money, for although money is in practice the almost universal numéraire in modern markets (except in some cases where rapid inflation may lead to the use of some other numéraire even though money is required by law to be used as a medium of exchange), this use of money as numéraire is in principle quite separate from its use as a medium of exchange. Indeed, in the perfect market which we are assuming there is no need for money as such: since transactions are costless, there is no advantage in the use of a specific circulating medium to reduce the number of transactions needed to convert the commodities that a trader wishes to supply into those that he wishes to procure, nor does any commodity command a liquidity premium over any other as being more acceptable or convenient as an intermediate commodity in a chain of exchanges.

A further assumption that will underlie a major part of the analysis is the assumption that the market is competitive. Traditionally, the competitive market has been interpreted as necessarily meaning a large number of buyers and sellers of each commodity, so that no individual buyer or seller could hope by his own independent action to influence significantly the price in the market; each trader would therefore behave as though the price were fixed beyond his control. Actually, what we are after here is merely an assumption that will justify the proposition that each trader will adjust his purchases and sales so that the marginal rates of substitution in terms of his own indifference map will be equal to the corresponding exchange ratios (at least for two commodities figuring in a ratio both of which are consumed in positive amounts; if one of the two commodities is not consumed, the appropriate inequalities must hold). For this purpose, it is not absolutely necessary that the individual traders in each commodity be numerous, if only they believe that they have no influence on the market price and act accordingly, even though in fact they might have a substantial influence. And even if they believe that they have some influence, but nevertheless determine to act as though they do not, from whatever motive, the results will be as developed on the assumption of competition.

On the other hand, merely the presence of numbers is no guarantee of competition, for collusion is always possible, and even in the absence of organized or overt collusion there may be any number of subtle influences at work tending to produce more or less substantial departures from competitive behavior. And obviously a single large trader may exercise an appreciable influence on price even though there may also be numerous small fry in the field. Strictly speaking, competition is an attribute of the behavior of buyers and sellers, rather than of the market itself, though that behavior may be greatly influenced by the characteristics of the market.

It is immediately apparent that administered prices do not normally conform to this concept of competition: one cannot be at once determining the price at which one will sell and assuming that prices are fixed beyond one's control. Perfect competition thus normally requires that each trader follow an "output policy" (or as a buyer, an "input policy") in which the amount to be placed on the market (or purchased) is determined in accordance with the price found to rule in the market, and is sold for whatever it will bring. A "price policy," in which a price is set after considering the state of the market and other factors, with the intention of selling as much as buyers wish to purchase at that price, is not an admissible policy if competition is to be perfect, though under favorable circumstances such procedures can produce results not too far removed from those predicted by perfect competition.

We have seen in the preceding section that the equilibrium of consumers

requires that they be at the highest point on what they consider to be their opportunity path, which, if they are behaving competitively, will be a straight budget line with a slope corresponding to the price ruling in the market. Equilibrium of the market in addition requires that the amount that buyers wish to buy at the current prices shall equal the amount that sellers wish to sell, for each commodity, so that there are no surpluses or unsatisfied demands. Fulfilling this condition in addition to the conditions of consumer equilibrium will ordinarily determine uniquely the equilibrium price ratios and the amounts bought and sold by each trader. To see how this comes about, it is convenient to study first some oversimplified cases.

The Two-Group, Two-Commodity Case

IN THE SIMPLEST possible case that is of any interest, we may consider an economy with two commodities, and two groups of traders each consisting of

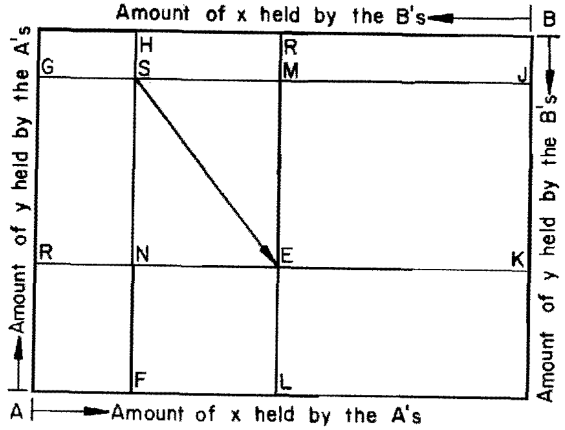

FIGURE 31

a number of individuals of identical tastes and identical initial resources, the number in each group being large enough to permit assuming competition. The two groups differ from one another, however, either in tastes or in initial resources or both, so that mutually advantageous trade between the members of the two groups is possible. Such a case can be illustrated in FIGURE 31, in which the width of the rectangle represents the total supply of one commodity, *x*, which we may think of as meat, and the height of the rectangle represents the total supply of the other commodity, *y*, say cloth. Any point within this rectangle will represent some division of this total supply between the *A*'s and the *B*'s; thus the point *S* represents a situation where the *A*'s have *GS* of *x* and *FS* of *y*, while the *B*'s have the remaining *JS* of *x* and *HS* of *y*.

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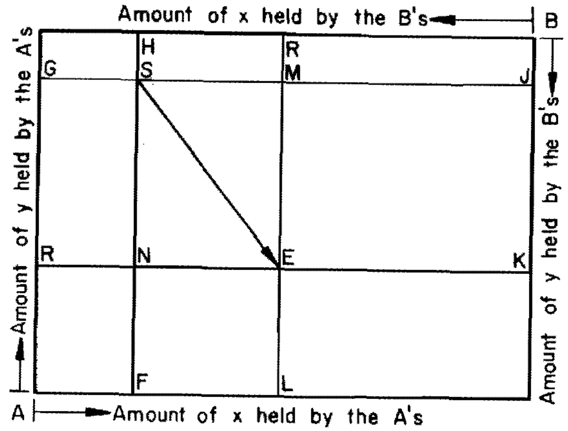

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An exchange may be represented on such a diagram by a motion from one point to another: thus going from *S* to *E* means that the *A*'s buy from the *B*'s an amount *SM* of *x*, and that the *A*'s pay the *B*'s an amount *SN* of *y*. The price of *x* in terms of *y* is then represented by the slope of the line *SE*, and the final situation is that the *A*'s have *RE* of *x* and *LE* of *y*, while the *B*'s have *KE* of *x* and *QE* of *y*. We assume that since they have the same tastes and the same opportunities, all of the individual members of the *A* group behave

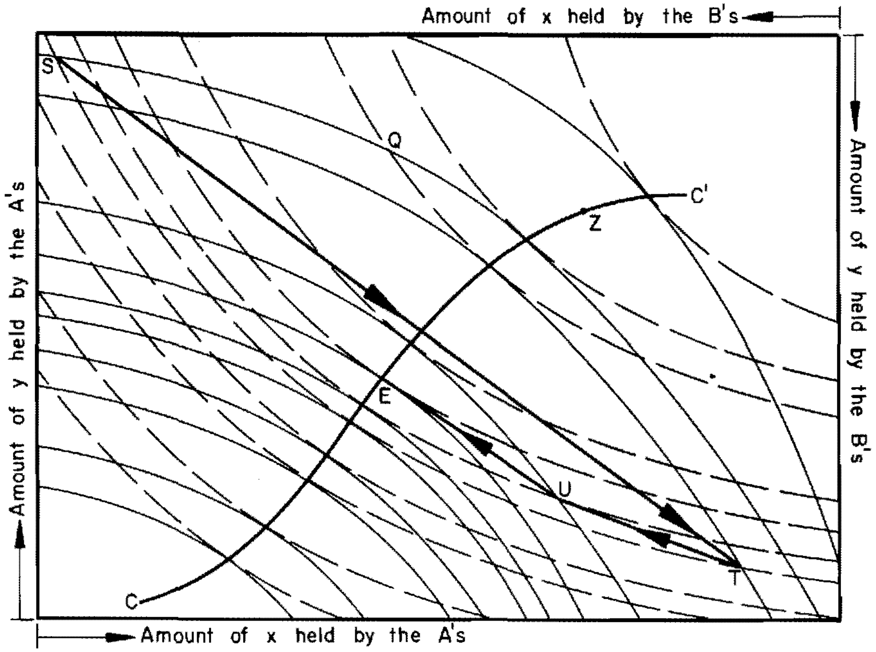


FIGURE 32

similarly, and likewise for the members of the *B* group; we can then consider the diagram as showing the aggregate shares of all the *A*'s and all the *B*'s, or as showing the shares of each individual *A* and each individual *B*, provided only that the scales of the diagram are changed in proportion to the number of *A*'s and *B*'s, respectively.

Considering the diagram as representing individual shares, we can superimpose upon the diagram the indifference curves of the individual *A*'s and *B*'s as shown in FIGURE 32 (*A*'s curves being shown by broken lines, *B*'s curves by solid lines). The summit of the *B*'s indifference map is near the origin of that of the *A*'s, where the *B*'s have most of the *x* and the *y* and the *A*'s very little; the summit of the indifference map of the *A*'s is in the top right-hand corner near the origin of the *B*'s map.

Through any point *S* there will in general be two of these indifference

curves, one for the *A*'s and one for the *B*'s, and these two curves will enclose a lens-shaped area which is the area of mutually advantageous exchange starting from *S*. That is, a move from *S* to any point *T* inside this area will be a move to a higher indifference curve for both the *A*'s and the *B*'s. The point *T* in turn will in general define a further somewhat smaller area of mutual advantage so that a further trade from *T* to, say, *U* inside this new lens-shaped area will be of advantage to both parties. Eventually, however, a point *E* is reached where the two indifference curves are tangent to each other, the lens-shaped area enclosed by the two curves has entirely disappeared, and there is no possible further trade that would benefit both parties. It is still possible to trade from *E* in a way that would benefit one at the expense of the other, but the one injured by such a trade normally has the power to refuse to trade further and at such a point trading ceases. The curve *CC′* passing through all such points where the indifference curves are tangent is known as the "contract curve."

In general, for any point *Q* not on the contract curve there will be a number of points on the contract curve that will be preferred to *Q* by both *A* and *B*; on the other hand, for any point on the contract curve it will in general be impossible to find any other point that does not make either *A* or *B* worse off. Thus if there is any allocation of resources that is to be considered an optimum it must be located somewhere along the contract curve. For if anyone were to assert that a point such as *Q*, which is not on the contract curve, represented the best possible allocation of resources, one could immediately point to some point on the contract curve within the two indifference curves passing through *Q*, such that both *A* and *B* would agree that it is preferred to *Q*; such a point *Q* could hardly be considered an optimum when there is another attainable point unanimously preferred to it, unless one were to bring into consideration some criterion extraneous to the preferences of the *A*'s and *B*'s.

This is not to say, however, that any point on the contract curve is necessarily to be preferred to a point not on the contract curve. For example, point *Z* might be held to represent such an extremely unequal distribution of resources as between the *A*'s and the *B*'s that the point *U*, though not on the contract curve, would be preferred as representing a more satisfactory or equitable distribution as between the *A*'s and the *B*'s. But if one is told that *U* is to be preferred to *Z*, then one can say that there exists on the contract curve a point *E* that is at all events to be preferred to *U*, and hence in this case also to *Z*. Without making interpersonal comparisons we can narrow the location of the point that is to be considered optimum to somewhere on the contract curve; determining which point on the contract curve is to be considered the most desirable requires making such interpersonal comparisons, or at least the setting up of some standard as to the proper distribution of income among the various members of the economic community.

Competitive Equilibrium

IN DEALING with a frictionless competitive market, of course, consumers do not make successive purchases at different prices, but rather are faced with a single price at which they can buy or sell as much as they want. To consider the behavior of the *A*'s and the *B*'s in such a market, we can draw their offer curves from the starting point *S*, as shown in FIGURE 33. The competitive market will be in equilibrium when the price is such that the amount the *A*'s wish to buy is equal to the amount the *B*'s wish to sell. This equilibrium point *E* is found at the intersection of the two offer curves, the price line *SE* determining the equilibrium price. Since the point *E* is on *A*'s offer curve, *A*'s indifference curve at *E* must be tangent to the price line *SE*; similarly, *B*'s indifference curve at *E* must be tangent to the price line *SE*, if *E* is on *B*'s offer curve. If the two indifference curves are tangent to *SE* they must be tangent to each other, and therefore *E* must be on the contract curve *CC'*. Thus competition produces a result that lies on the contract curve. It is therefore not possible to move away from the competitive equilibrium in a way that will benefit all parties, or benefit some parties and injure none. This

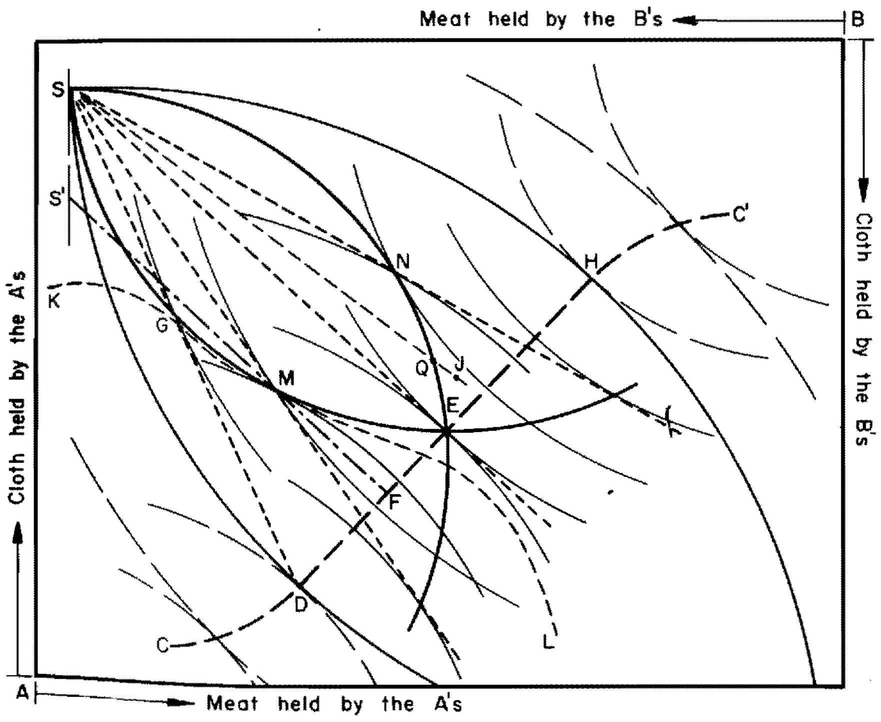


FIGURE 33

is the fundamental justification for the claim that competition produces an optimum allocation of resources.

Simple Monopoly

THE COMPETITIVE result may be compared with other possible situations. For example, let us suppose that the *B*'s get together and organize a joint trading agency in the operations of which we may suppose that they are all to share equally, and that this trading agency is able to fix the price at which it will sell *x* for *y*, while the *A*'s accept this price as a datum and buy competitively as indicated by the offer curve. Alternatively, and perhaps with less disturbance to the formal operation of the market, the *B* agency might decide on the total amount of *x* to be sold, allowing the bidding of the *A*'s on the market for this quantity to determine the price. We will assume further that the *B* agency is able to determine, by experiment or otherwise, the position of the offer curve of the *A*'s. By thus determining either the supply or the price, and letting the *A*'s then determine the corresponding price or quantity, the *B* agency can choose among the points that lie on *A*'s offer curve. Thus the offer curve of the *A*'s becomes the opportunity path for the *B*'s. This is a case where the opportunity path is a curve, in contrast to the straight-line opportunity path that obtains when price is considered fixed. If the *B* agency sets the price (or the quantity) so as to reach the highest possible point on the indifference map of the individual *B*'s, it will select the point where the offer curve of the *A*'s is tangent to one of the indifference curves of the *B*'s, i.e., at *M*.

This would be, incidentally, one case where it would be possible to observe the choice by a consumer of a point within a concavity in his indifference map, as would be the case, for example, if one of the indifference curves of the *B*'s had the shape indicated by the curve *KL*. It is one thing, however, to observe a consumer at a point that may in fact be in a concavity, and quite another to show from a series of observations that it is a concavity. Indeed, while an observation of a competitive equilibrium at *E* tells us immediately that the slope of the indifference curve at *E* is equal to that of the price line *SE*, observation of monopoly at *M* itself tells us nothing about the shape of the indifference map, unless we happen to know the offer curve of the *A*'s. And indeed, it is not the actual offer curve of the *A*'s that is relevant, but what the *B* agency considers the offer curve of the *A*'s to be. This great difficulty in ever finding concrete evidence of the existence of such concavities in terms of overt behavior can be considered further ground for disregarding such possibilities in most economic analysis.

The point *M* arrived at by the monopoly is not in general on the contract curve. This does not necessarily mean that the point *M* is to be considered inferior to the point *E*. Indeed, if the *B*'s initially had the smaller share of the total resources, then changing from competition to monopoly on the part of the *B*'s might be approved as representing a method of redressing this originally unequal distribution. But while granting that this monopoly might be considered better than leaving the original competitive situation alone, it is certainly not the best way of redressing this inequality. For there exists a point *F* on the contract curve that is unequivocally better than *M*. Rather than permit the *B*'s to organize the monopoly, it would be better to redistribute the initial resources from *S* to *S'* by some system of taxes or bounties, and then competition would produce a result on the contract curve in the vicinity of *F*. Or if a monopoly by the *B*'s is established at *M*, it would then be to the advantage of the *A*'s for them to make a lump payment of *SS'* to the *B*'s, if they could do so, in return for assurance that the *B*'s would give up their monopolistic practices; such a procedure might likewise result in *F*, or at least a point on the contract curve in the neighborhood of *F*, which would be preferred to *M* by all concerned.

Discriminatory Monopoly

ANOTHER possibility would be that the *B*'s practice not mere simple monopoly in which they set a single price and allow the *A*'s to take as much or as little as they want at that price, but instead practice discriminatory monopoly in which they sell different amounts under different circumstances at different prices. The most extreme and at the same time the simplest form of such discriminatory monopoly is for the *B*'s to offer each *A* the opportunity to buy a set amount at a set price on an all-or-nothing, take-it-or-leave-it basis. In this case, the *B*'s will derive the greatest possible advantage by offering the *A*'s a trade that will bring them to point *D* on the contract curve, just inside the indifference curve of the *A*'s that passes through *S*. The *B*'s can in effect select for the final result any point inside the area of mutual advantage, but not one beyond this, for in that case the *A*'s, finding themselves better off at *S* than at the proposed point, will simply refuse to trade at all. If the *B*'s could enforce on the *A*'s a trade that was to the net disadvantage of the *A*'s, then the situation would be considered no longer one of trade but one of tyranny.

Opportunities for discrimination of this type are comparatively rare, however. To begin with, it is necessary that the *A*'s be unable to resell among themselves, for if reselling were possible, then it would be possible for, say, half of the *A*'s to accept the offer *SD*, and then share with the remaining half of the *A*'s, thus arriving finally at *G* rather than at *D*, which might, as in FIGURE 33, be even less desirable to the *B*'s than the results of simple monopoly at *M*. Opportunities for such discrimination arise chiefly with respect to personal serivces such as medical care, hairdressing, or transportation, which once performed cannot be transferred. Moreover, while here we

have assumed that the tastes of all of the buyers were the same, so that the same offer would extract the maximum advantage from each, in practice there would be variations among the buyers as to tastes and resources, so that a separate appraisal of the situation and a separate offer would be required for each of the buyers if the point *D* were to be attained; this would be extremely difficult to achieve. Thus while a theoretically perfect degree of discriminatory monopoly would produce a result on the contract curve, in practice a result close to the contract curve may be considered quite unlikely.

Types of Equilibrium

WHILE *E*, *M*, and *D* are all in a sense points of equilibrium, the nature of that equilibrium is quite different in the competitive case from what it is in the monopoly cases. The equilibrium at *E* is reached more or less automatically through the market mechanism. Each individual concerned reacts only to a price that is determined in the market and that is an overt fact of common knowledge, in combination with his own preference schedule concerning which he himself is the final judge. In the case pictured in FIGURE 33 the equilibrium at *E* is a stable one, as can be seen from the similar case shown in FIGURE 34, in which the offer curves of the *A*'s and the *B*'s are shown in relation to the corresponding demand and supply curves. If the price should happen to be greater than the equilibrium price for some reason, as represented by the price line *SLK*, or the ordinate *oq*, then the amount the *A*'s wish to buy, as indicated by the offer curve, will be *SU* (or *ql* as indicated by the demand curve); the amount the *B*'s wish to sell as indicated by the offer curve of the *B*'s will be *SW* (= *qk*). In this situation, some *B*'s who wish to sell will be unable to find buyers, and their presence in the market unsuccessfully seeking buyers will drive the market price down toward *op*, and the price line toward *SE*. Similarly, if for any reason the market price were below the equilibrium price, the presence of unsatisfied buyers would tend to push the price up and restore the equilibrium situation.

But it should not be assumed that any intersection of the offer curves represents necessarily a point of stable equilibrium. There is in fact nothing to prevent the offer curves from intersecting several times, as in FIGURE 35. In such cases there will be a corresponding number of intersections of the demand and supply curves. The intersections will represent alternately stable and unstable equilibria. In FIGURE 35, the points *E*, *F*, and *G* represent stable equilibria, while *U* and *V* represent unstable equilibria. As long as the price remains at the point *U* or *V*, supply and demand are equal and there is nothing to produce an immediate change. But any slight movement away from these points caused by any random disturbance will bring into play forces tending to produce further motion in the same direction, away from

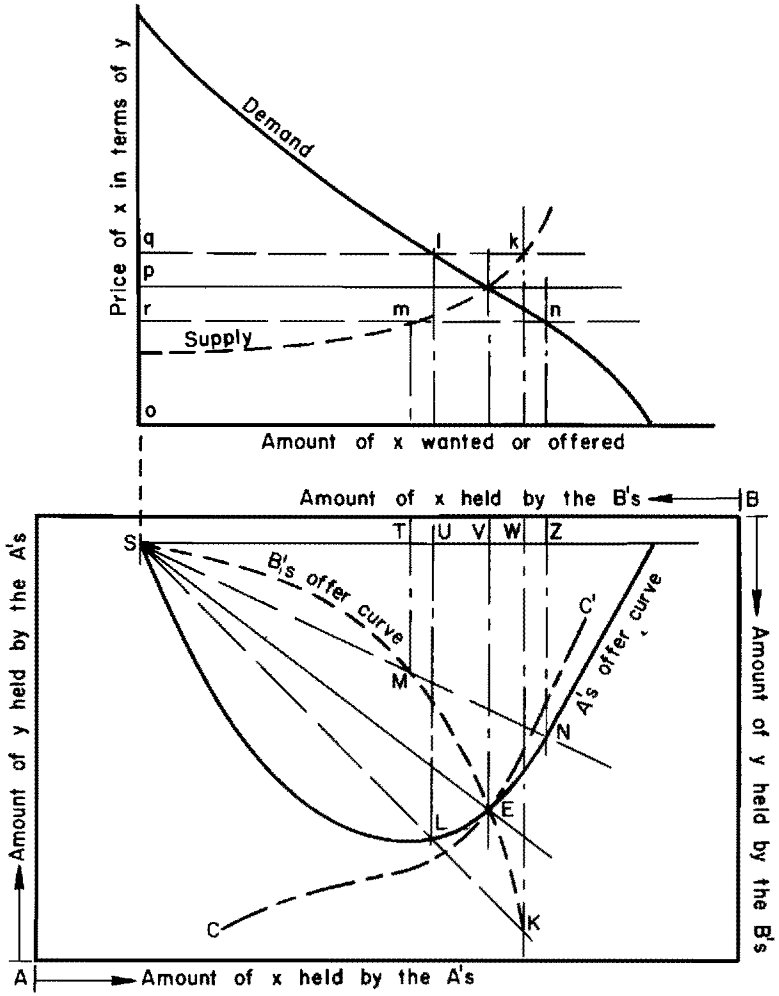


FIGURE 34

U or *V*, and this shift will continue until one of the points of stable equilibrium is reached. Thus if the price should rise above either *u* or *v*, demand would exceed supply and the presence of unsatisfied buyers would push prices up still further until the market finds another equilibrium at *f* or *g*, and conversely for a fall in price. The point *F* may be termed a point of micro-stable equilibrium in that if disturbances are kept sufficiently small, the market pressures resulting from the relation between demand and supply will tend to bring the price back to the equilibrium at *F*. But if there is a disturbance sufficiently large to carry the price below *u* or above *v*, the price will tend to continue on to *e* or *g*, respectively, and the situation *f* will not be regained.

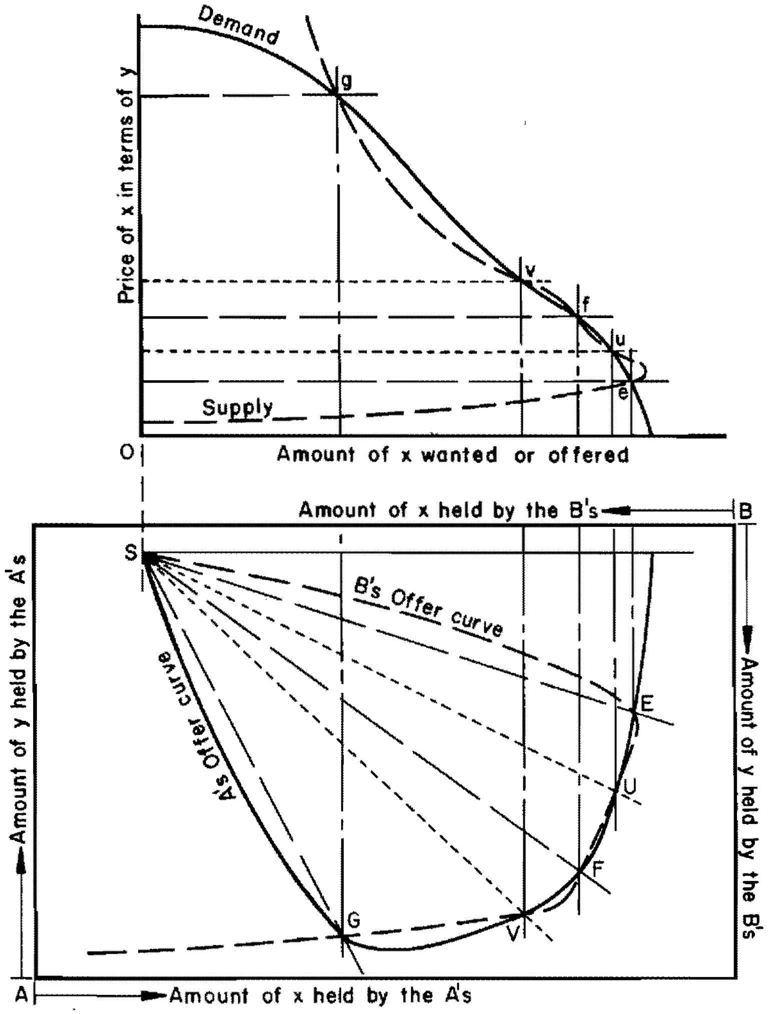


FIGURE 35

Similarly, point *e* is stable against downward variations in price of any magnitude, but merely micro-stable against upward fluctuations in price, and likewise, *mutatis mutandis*, for point *g*.

Another possibility is illustrated in FIGURE 36, in which we have neutral equilibrium over the range between the points *NN'*. At any point in this range, buyers and sellers are doing all the trading they wish to do at the going price, and there are no unsatisfied buyers or sellers tending to push the price up or down; if a change in the price within this range is brought about by some external or accidental force, there is nothing in the system as stipulated that would tend to restore the initial situation. Just where in the range *NN′* the situation will stand at any given moment will depend on chance or on outside factors such as bargaining power, historical accident, institutional factors, or other similar influences. Indeed, the moment we abandon the assumption of strictly competitive behavior on the part of the traders, the situation may be thought of as violently unstable, since any small reduction in demand or supply would tend to drive the price down to *N′* or up to *N*, and thus each

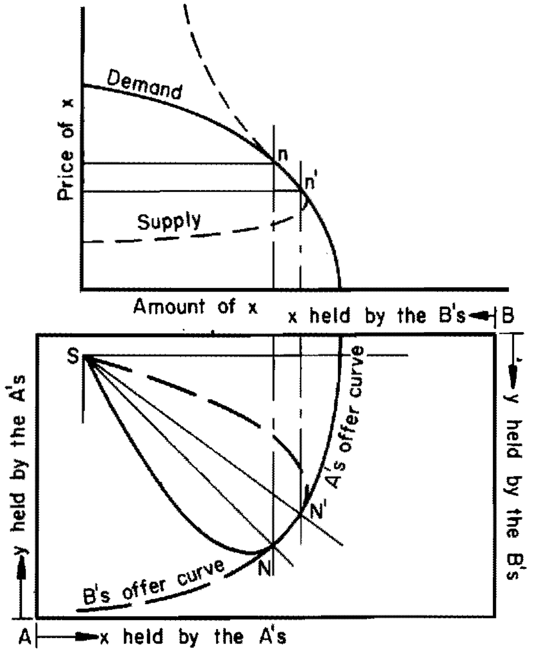


FIGURE 36

trader, even if only a small part of the total market, would have an incentive to withhold a part of his trade in the hope of influencing the price substantially in his favor. If all traders behave in this way, however, the effects may partially cancel out, and the results become difficult to determine.

In strictest terms, the occurrence of such cases of neutral equilibrium is of course infinitely unlikely a priori, since it requires the coincidence of two curves that are determined independently. However, if we extend this case to cover instances where the two offer curves are merely so close together that the market forces are insufficient to overcome frictions and inertias of various descriptions, this case may be of significant frequency, though probably still relatively rare.

Often the cases of multiple equilibria are dismissed as curiosa unlikely to be encountered in practice, but it is not at all clear that this is justified. Indeed, it

is possible to show a priori that given the total supply of *x* and *y* and the tastes of the *A*'s and the *B*'s, it will in general be possible to find starting points for which there will be multiple competitive equilibria. For the indifference maps and the aggregate supply determine the contract curve. At any point *E* on the contract curve, one can draw the common tangent to the two indifference curves through that point, and every starting point on such a tangent will have an equilibrium point at *E*. Now if we take any two points on the contract curve, and draw the tangents to the indifference curves at these two

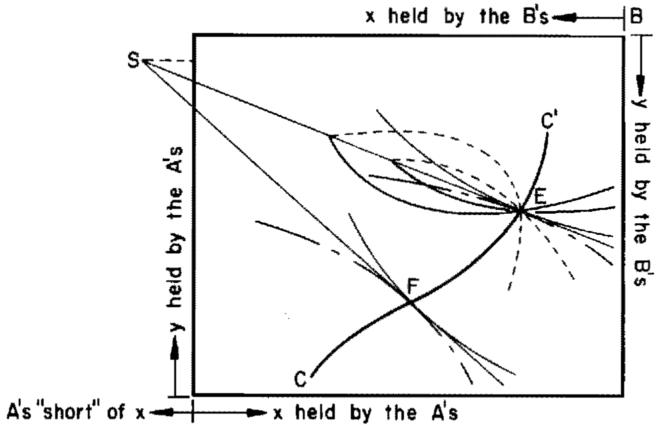


FIGURE 37

points, it is a priori unlikely that these two tangents will be exactly parallel, and still more unlikely that all such tangents at all the points on the contract curve are parallel. If the tangents at some two points *E* and *F* are not parallel, they meet at some point *S*, and if *S* is used as a starting point, then both *E* and *F* will be points of equilibrium, as illustrated in FIGURE 37. To be sure, the starting point so determined may lie outside the rectangle of positive resource distributions, but this may be interpreted in terms of having the *A*'s or the *B*'s or both start out owing the other some of commodity *x* or *y*; i.e., they may be considered as having entered into a "short" contract. This, to be sure, is a somewhat artificial situation; but on the other hand it would be rather presumptuous to assert that no two of all of the tangents at the contract curve intersected within the rectangle.

It is possible indeed to construct cases where no stable equilibrium exists at all. If we consider a case where the *A*'s start out with less than the minimum requirement of a necessity *x*, while the *B*'s similarly start with less than the minimum essential amount of *y*, so that neither could survive without trade, then we could conceivably have the situation depicted in FIGURE 38, where *U* is a point of unstable equilibrium, and where in the usual sense there are no other intersections of offer curves. If the price should rise above *u*, demand

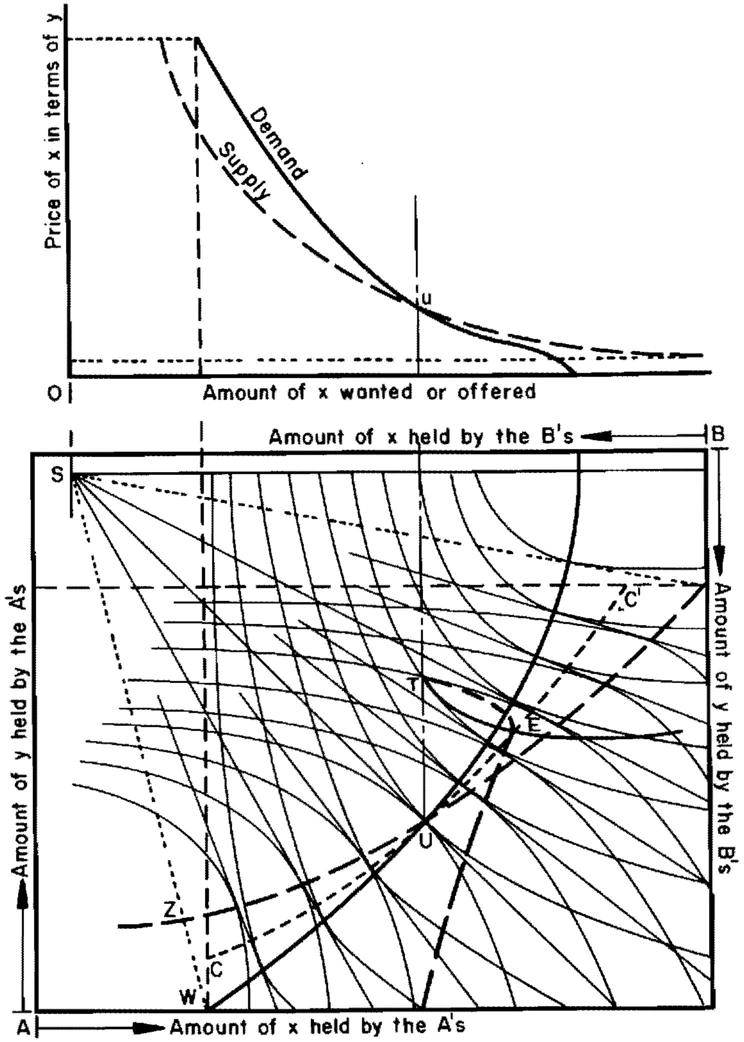


FIGURE 38

would exceed supply and drive the price still further up until the *A*'s must offer all of their *y* (or perhaps all of their *y* above the minimum necessary amount) in order to get enough *x* to subsist, as at *W*. But at this price the amount the *B*'s are willing to supply is still not enough to support all the *A*'s, there is still no equilibrium, and the price tends to go still higher. At such a higher price, the *A*'s are unable to get enough *x* to survive. If a sufficient number of the *A*'s die off, emigrate, or otherwise remove themselves from the picture, a solution of sorts may be found at *Z* relative to the indifference map

of the *B*'s, where the price is determined by the line *SW*, and the trade *SZ* is divided among a sufficiently smaller number of *A*'s so that the share of each in *SZ* is equal to the amount *SW* divided among the former larger number, and the reduced number of *A*'s is able to eke out a subsistence. This example might be considered a modern version of the "iron law of wages."

The tendency to exploitation in such a situation can be perfectly symmetrical: if, "come the revolution," the *A*'s manage to push the price below *u*, it will then be the *B*'s turn to cling to the margin of subsistence while the *A*'s enjoy the bulk of the available resources. Indeed, precisely this reversal might be considered implicit in the diagram if, by chance fluctuation, the offer curve of the reduced number of *A*'s gets pulled entirely inside the offer curve of the *B*'s in the neighborhood of *Z*, as might happen if the *A*'s, for example, suffer an epidemic that reduces their number below the number that can be supported at the subsistence level.

One can also read into this analysis a suggestion that specialization may be an important factor in producing instability. If, instead of starting out at *S* with the *A*'s specialized in producing *y* and the *B*'s specialized in producing *x*, they started at *T* where the *A*'s produce enough *x* to satisfy at least their own minimum requirements and the *B*'s likewise are the original producers of enough *y* for their own minimum needs, then the two offer curves intersect in the normal way at *E* and there is a stable equilibrium. If, then, seduced by the doctrine of comparative advantage, the two groups specialize, moving the starting point in the direction of *S* while at the same time expanding total output and shifting the indifference curves of the *B*'s relative to those of the *A*'s (a feature that cannot be clearly shown on the diagram, as the diagram is based on a fixed total output), an equilibrium such as *U* might be reached that is indeed unequivocally preferred by both groups to the original point *E*. But if the equilibrium at *U* breaks down, one or the other group may come to regret the trend to specialization, particularly if the shift has been irreversible so that it is not possible to return to the original situation at *T*. If the specialization process is reversible, then of course the exploited group would be able to some extent to protect themselves against the more extreme results by shifting their production back toward diversification.

At this point it should be mentioned that for a demand curve to lie to the right of a supply curve at prices lower than the equilibrium price (as at *g* in FIGURE 35) does not always indicate a stable equilibrium, nor the reverse (as at *v* in FIGURE 35) an unstable one; each case has to be examined in terms of the nature of the market being represented by the curves and the precise meaning attached to the demand and supply curves themselves. For example, in FIGURE 39 the relative position of the demand and supply curves is apparently similar to that at the points *u* and *v* in FIGURE 35, and might, accordingly, be thought to indicate an unstable equilibrium. But consider the case where the supply curve *S*, in FIGURE 39, rather than representing the response of sellers to a market price assumed to be fixed, instead represents a declining long-run average-cost curve in a competitive industry in which there are external economies of scale, so that it represents the level to which the price that will be charged will settle, given the total amount that can be sold. Here, if price exceeds the equilibrium price, then demand exceeds supply; this may push the price up, temporarily, to be sure, and in the short run there may be an equilibrium determined by the intersection of the short-run supply curve S' with the demand curve D; but eventually new firms enter the industry, or old firms expand their capacity and drive the price down again, and the external economies permit the price to be reduced below its original level,

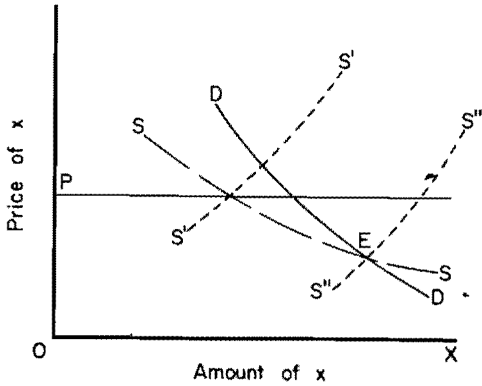


FIGURE 39

until finally the long-term equilibrium is reached at E. This will be explained more fully in Chapter 6. Conversely, if the positions of the demand curve and the long-run average-cost curve are interchanged, the intersection would be a point of unstable equilibrium, and not of stable equilibrium as is the case with the similar-appearing point g in FIGURE 35.

This is but one of the pitfalls that lurk in the way of a determination whether a given equilbirium is stable or otherwise. Indeed, this question cannot be fully answered without a lengthy investigation of the processes by which an adjustment of a disequilibrium resulting from a disturbance takes place, and such investigations are often not complete without fairly extended excursions into the realm of dynamics. The problem moreover becomes much more complicated when the analysis is extended to more than two commodities: a single price fluctuating in the neighborhood of the equilibrium is very likely to pass through the equilibrium point in its movements; a system of two or more prices, however, can easily keep swinging in circles around the equilibrium point without ever reaching a situation where all prices are at their equilibrium levels at the same time, so that disturbances continue. And a system may well be stable with respect to disturbances in one price, or one

parameter, at a time, but be unstable with respect to combined disturbances in two or more parameters. Great caution is therefore necessary in interpreting the results of static analysis, and particularly of analysis limited to two dimensions at a time.

The nature of the monopoly equilibrium is quite different from that of the competitive one, and it too bears examination. The competitive equilibrium is brought about by the actions of individuals based on their own preferences on the one hand and the market prices on the other; the market price is an objective fact concerning which they are not likely to remain ignorant. The

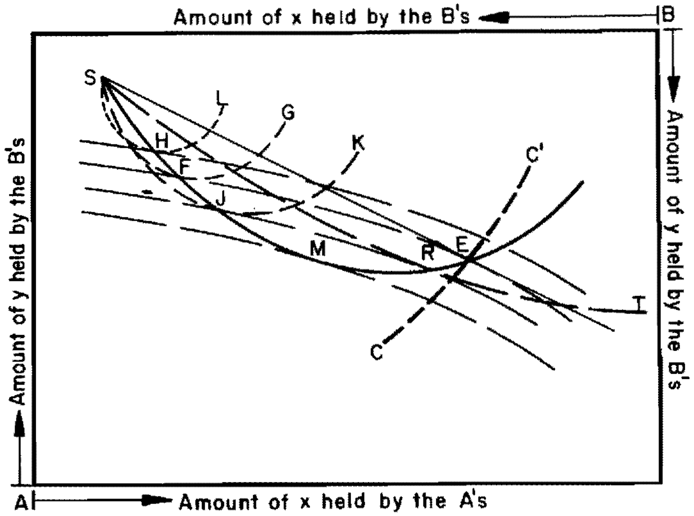


FIGURE 40

monopoly equilibrium, on the other hand, is the result of the monopolist's considering his own preferences in relation to *what he believes to be* the offer curve (or demand curve) of the buyers. Now there is no reason to suppose that a monopolist will necessarily have a correct impression of this offer curve. All that his current experience tells him unequivocally is that a given point at which he is operating is on the offer curve of the buyers, and he may be completely wrong about the direction or slope of the offer curve at that point, without this error giving rise to any apparent inconsistency. The price actually set by the monopolist may therefore be not that which actually produces the greatest satisfaction to him of the points on the actual offer curve, but merely a point on the actual offer curve that produces a greater satisfaction to the B's than any other point on the curve that the B's *believe to be* the offer curve. For example, if in FIGURE 40 the curve $SJMRE$ represents the actual offer curve of the A's, then the equilibrium point is M if we assume that the monopolist estimates this offer curve correctly. However, it would be perfectly possible for the monopolist to set a price that would result in position *J*, and to be satisfied to remain there because he believes that the offer curve is *SJK* rather than *SJMRE*. Or the monopolist might be satisfied with setting a price that results in *R*, under the impression that the offer curve is *SRT* rather than *SJMRE*. At *J*, both the *A*'s and the *B*'s are worse off than they would be at *M*; at *R* the *B*'s are worse off but the *A*'s are better off than at *M*. In effect, an error by the *B*'s that carries the equilibrium point toward *S* from *M* injures both parties; an error from *M* in the direction of *E* will benefit the *A*'s at the expense of the *B*'s. It is thus in the interests of the *A*'s to propagandize the *B*'s to the effect that their offer curve is like *SRT* rather than *SJK*, i.e., that their demand is elastic. In any case, the result will still be off the contract curve: the only assumed offer curve that would lead to the *B*'s selecting the point *E* would be a straight line *SE*, implying that the demand is perfectly elastic. But no monopolist acting as such would assume a perfectly elastic demand, or at least if he did, he would be acting, *pro tanto*, in a perfectly competitive manner.

To be sure, the monopolist might experiment with different prices so as to actually trace out a portion of the offer curve. However, such experimentation takes time, if only because consumption does not respond instantaneously to price changes (or at least the reaction to a temporary price change may be different from that to a more permanent change, owing to anticipatory buying and the like). Further, if we conceive of other monopolists making similar experiments in fields that impinge on his, additional sources of uncertainty are introduced. And of course in the real world the over-all situation is continually changing, so that there seems to be no way of assuring that the monopolist's estimate of the offer curve will ever approach the correct one, even allowing an extremely long period for the process of adjustment to work itself out without the injection of further exogenous disturbances.

A somewhat similar condition surrounds the equilibrium at *D* that the discriminating monopolist is conceived to be attempting to approach. All that the monopolist will know for sure is that the offer is or is not accepted and is therefore inside or outside the area of mutual advantage determined by the starting point *S*. And since the various methods of discrimination and the various possible combinations of rates involve a large number of parameters each one of which may be varied, instead of only a single price, the possibility of exploring substantial proportions of the likely possibilities is even more remote. The likelihood of approaching the point *D* with any great degree of precision must be considered even more remote than that of arriving at the point *M*.

The case where the *A*'s as buyers of *x* organize a monopsony while the *B*'s act competitively is of course exactly symmetrical, the differences being solely those of terminology.

Bilateral Monopoly

THE SUPERIMPOSED indifference-curve diagram of FIGURE 33 can also be used to study the case of bilateral monopoly in which there are only two parties involved; in fact, this appears to be the problem to which this diagram was first applied when it was developed by Edgeworth. But there is on the whole very little that can be said with certainty about the outcome of such a case, other than that any trade must proceed to a point within the area of mutual advantage. Early economists supposed that if the bargaining process lasted long enough one could assume that one would reach a final contract located on the contract curve (hence its name), somewhere between *D* and *H*, the exact point depending on the relative bargaining abilities and strategic strength of the two parties. This would seem, however, to assume that having made one deal, from, say, *S* to *T* in FIGURE 32, the way would still be open for a further trade from *T* to *U*, which being of advantage to both parties would necessarily be made. Possibly this could be justified on the basis of a latent assumption that the particular occasion is unique, and that neither party has in the back of his mind the effect of his current action on the type of trade made on the next occasion. Indeed, one party might refuse to agree to a trade from *T* to *U* on the ground that to do so might prejudice his opportunity to get an initial trade as favorable as *ST* in the next succeeding period. Or negotiations may be stalled through each party holding out in the hope of compelling the other to accept a more favorable settlement: the parties may remain in this way at *S* for more or less protracted periods of time, as is observed in the case of strikes.

To be sure, if at least one party is supposed to know the indifference map of the other, as might happen if negotiations are conducted in an open and cooperative spirit, or if the nature of the needs and requirements of one of the parties is fairly obvious to the other (as when the nature of a manufacturer's operations is fairly well understood by labor-union leaders), then it would be apparent to at least one of the bargaining parties that a move, from *T* to *U* in FIGURE 32, for example, would be advantageous to both, and thus one could expect in such cases a result close to the contract curve. In cases where the bargaining parties are very similar in nature, there will be an agreement to split the benefits derived from the trade more or less equally between the parties, thus landing fairly close to the contract curve about half way between *H* and *D*; such a point may in many cases be close to *E*, the competitive equilibrium, but need not be. Contracts for the interchange of power between interconnected electric-power systems are often of this type, for example.

However one's ability to conceal the nature of one's indifference map from the opposite party, or even to mislead him as to its shape, is often one of the important elements in bargaining power. If neither knows the indifference map of the other, then neither knows where the contract curve is, and it is possible for trading to stop at T with each party believing that the contract curve has been reached, or at least with each being uncertain, and perhaps of opposite opinions, as to the direction in which the area of further mutual advantage lies. And the nature of the bargaining process may be such as to preclude their mutually enlightening each other. There is here a basic unsolved problem of prescriptive economics: Is it possible to devise a set of bargaining procedures that in the light of this analysis would be most likely to lead to an approach to the contract curve without prejudice to the interests of either party?

The contracts resulting from bilateral monopoly very often take the form of the establishment of a price with one or the other party free to determine the scale of the transaction, rather than take the form of a specification of the quantities to be traded. Thus labor contracts frequently specify the wage rate, with the employer free to hire as much labor at that rate as he wishes (or, in some cases, as much as he can find). In such cases, the locus of possible final contracts that suggests itself consists of the two segments of the offer curves ME and NE in FIGURE 33. As bargaining power is normally exercised by buying or selling less than one would under competitive conditions rather than more, the portions of the offer curves beyond the contract curve (as seen from S) seem to represent unlikely results, as do the portions of the offer curves short of the monopoly and monopsony points.

Even where the contract is to specify both amount and price, there appears to be some indication that each party will feel that in the bargaining process the price will be influenced in his favor by his offering to buy or sell less; accordingly, if one party were to consider a possible contract J in the region between the offer curves and the contract curve, he would in general be better off at the point Q on his offer curve involving the same price, and since pushing for Q would not be likely to move the price against him, not much consideration would be given to J as a possible contract. Such hazy considerations lead to a notion that the area between the offer curves, and especially the area just inside the portion MEN, is perhaps somewhat more likely to contain the final contract than other areas in the entire zone of mutual benefit. But such notions are highly speculative.

Other attempts have been made to derive rational results in this situation on the basis of game theory; however, most of the analysis of game theory requires the assignment of cardinal utility values to the indifference curves, and is generally too abstruse to be considered here; accordingly, it is deferred to the section on imperfect competition generally.

Other Uses of the Bargaining Diagram

THIS TYPE of indifference-map analysis is sometimes applied to problems of international trade. If we are willing to assume that two nations are composed

of identically situated individuals with identical tastes, the analysis in terms of the groups of A's and B's applies directly; the only difficulty is that the introduction of such things as transportation costs, tariffs, and the like tends to make the diagram somewhat more complicated and difficult to use than it is where used to represent simple trade in a perfect market. To apply the analysis to cases where tastes and resources vary among the members of a nation, it is necessary to find a way of defining something that can be described as the indifference map for the nation or community as a whole. If one can assume that within each nation the distribution of goods is optimalized through a perfectly competitive market, and that redistributive devices can be employed to maintain any specified distribution of incomes among individuals, then *relative to a given distribution of income* a community indifference curve can be defined as the locus of those minimal combinations of goods and services that if optimally distributed would suffice to keep all members of the community on the same indifference curve as they found themselves in the initial state. The fact that changes in the international trade situation will almost inevitably produce changes in the internal distribution of income within a country and hence change the community indifference map vitiates the exactness of this tool of analysis to a greater or less extent; nevertheless, in some contexts it can be used as an approximate instrument, provided that its limitations are kept firmly in mind.

Another method of constructing a community indifference map is merely to infer such a map from observed or presumed amounts supplied and demanded in the aggregate at various prices, assuming a competitive aggregate behavior. But while transitivity of choice is not too unreasonable an assumption for an individual conceived to have a consistent set of preferences, transitivity is much more difficult an assumption to justify in the case of a community made up of individuals with conflicting interests; nor is it clear what welfare implications could legitimately be drawn from such a map without fairly drastic assumptions as to constancy of income distribution and the like. Used with care, however, such indifference maps provide a useful first-order approximate analysis in many situations.

The process of exchange can also be appraised in terms of maximizing the combined consumers' and sellers' surplus. If the indifference maps of both parties are such as to admit of the assumption of a constant marginal utility of money or of the numéraire commodity, then the contract curve becomes a vertical straight line parallel to the money axis, and any point on the contract curve will maximize the sum of the consumers' and the sellers' surplus. This maximum sum will vary according to the starting point (it will obviously be zero if the starting point is itself on the contract curve), but for any given starting point all points on the contract curve would yield the same sum, equal to the distance between the two points at which the indifference curves through the starting point intersect the contract curve. If, however, the indifference maps do not admit of such an assumption, then there is relatively little that can be said in this direction, other than the rather trivial proposition that some point on the contract curve will yield a higher sum of the equivalent variations (from any given starting point) than any other point lying between the two indifference curves passing through the given point on the contract curve. The sum of the surpluses will indeed vary, in general, as we move along the contract curve, but there would appear to be very little virtue in selecting a point on the contract curve that maximizes one variety or another of the consumers' and the producers' surplus.

General Equilibrium of Exchange

ALL OF THE ABOVE has to do with an extremely simplified economy of only two goods and two types of traders; nevertheless, the results are quite interesting and give considerable insight into the workings of more complicated systems. However, to extend the same type of analysis to more complicated systems involving more commodities or more persons immediately goes beyond the capacity of mere two-dimensional diagrams. To be sure, one can conceive of a corresponding diagram for a three-commodity two-party system, in which a point located within a three-dimensional rectangular prism would represent the division of the three commodities between the two parties; the contract curve is still a one-dimensional curve running from one corner diagonally to the opposite corner; indifference curves become indifference surfaces, and offer curves become offer surfaces. Some new methods of analysis have to be applied by reason of the fact that the offer curves meet in a curve rather than a point so that the equilibrium point cannot be determined merely from the intersection of the offer surfaces, and a trade from one point to another on the diagram can in general be accomplished under more than one set of prices. But very little that is essentially new is added by going into these details. And if we go to the case of three parties and two commodities, four dimensions are immediately required for a complete representation.

Actually, very little can be done with the more complicated cases without introducing a certain amount of mathematical notation, and once this is introduced, it causes very little additional difficulty to proceed at once to the general case of simple competitive exchange of R commodities among N individuals. In such a more general case, it is not possible to say very much in a simple fashion about the patterns that might develop; about all that can be done is to show that a system in which all the elements are interrelated may still have a definite equilibrium state, and to ascertain just how much information is in principle necessary in order for the equilibrium state to be determinate.

In examining the general case of competitive exchange, we take as given the

indifference maps or preference systems of the individual traders and the amounts of the various commodities with which each trader comes to market. For the simplest case where at the equilibrium point each trader consumes some positive amount of each commodity, we can then consider that equilibrium requires (1) that for each trader the marginal rate of substitution between any two goods for the point on his indifference map corresponding to his final consumption of goods must be equal to their ratio of exchange in the market; (2) that the purchasing power represented by the goods which each trader brings to the market must be just exhausted by the final consumption of each trader (i.e., each trader takes away from the market as much as he can); and (3) that the total amount of each commodity taken away from the market by all traders must equal the total amount brought to the market: demand must equal supply. It turns out that these conditions are just sufficient in general to determine a solution, and that on the other hand a solution can in general be found that fulfills all these conditions.

Let us put $q_x{}^a$ for the amount of commodity x consumed by individual a,

$$S^a_{y/x} = S^a_{y/x}(q_x{}^a, q_y{}^a, q_z{}^a, \ldots q_r{}^a)$$

for the marginal rate of substitution of y for x for individual a, which is assumed to depend only on the amounts of the various commodities consumed by a;

$$U_x{}^a = U_x{}^a(q_x{}^a, q_y{}^a, q_z{}^a, \ldots q_r{}^a)$$

for the marginal utility of x to individual a according to an unspecified utility index, assumed likewise to depend only on the amounts of the various commodities consumed by a; and $p_{x/y}$ for the price of x in terms of y. Then for each individual the first condition gives us a series of relations such as $S^a_{y/x} = p_{x/y}$. Since we know that whatever index is arbitrarily selected as a measure of utility, we must have $S^a_{y/x} = U_x{}^a/U_y{}^a$, and since $p_{x/y} = p_x/p_y$ regardless of what money or numéraire the prices p_x, p_y, etc., are expressed in, we can write $p_x/p_y = U_x{}^a/U_y{}^a$, or turning this proportion the other way we can write $U_x{}^a/p_x = U_y{}^a/p_y$. Combining this with similar relations between other commodities we can combine all the marginal conditions for the individual a in the following set of relations

$$\frac{U_x{}^a}{p_x} = \frac{U_y{}^a}{p_y} = \frac{U_z{}^a}{p_z} = \frac{U_w{}^a}{p_w} = \ldots = \frac{U_r{}^a}{p_r} \qquad (1)$$

which is to say that for each individual a, the marginal utility of a dollar's worth of each commodity must be the same as for any other commodity. If there are R commodities, there will be in effect $R - 1$ independent equations of this kind for each individual, for if we count the equation between x and y, for example, and also that between y and z, it is not possible to consider an equation between x and z as an additional independent equation, for it gives us no new information not contained in the previous two, since it can be derived from them.

If there are thus $R - 1$ of these equations for each individual, there will be altogether $(R - 1)N$ of these equations for all the N individuals.

Let us further put Q_x^a for the amount of x bought by individual a—i.e., the excess of the amount q_x^a consumed over the amount r_x^a he started out with. If a happens to be a supplier rather than a purchaser of x, we can express the amount supplied as a negative purchase. Thus if a started out with 25 pounds of x, and consumes ten pounds, we would have $Q_x^a = -15$ pounds. The second or "budget" condition for consumer a then becomes

$$Q_x^a \cdot p_x + Q_y^a \cdot p_y + Q_z^a \cdot p_z + \ldots + Q_r^a \cdot p_r = 0 \qquad (2)$$

That is, the sum of the value of the goods bought less the value of the goods sold must be zero. There will then be one of these budget equations for each consumer, or N budget equations altogether.

Finally, the third, or "market-clearing," condition may be expressed as follows:

$$Q_x^a \cdot p_x + Q_x^b \cdot p_x + Q_x^c \cdot p_x + \ldots + Q_x^n \cdot p_x = 0 \qquad (3)$$

That is, the sum of the values of the purchases of x by the various consumers less the value of the sales of x by the various sellers must be equal to zero. This equation is sometimes written without the common factor p_x in each term, but it simplifies the exposition slightly to leave it in. There will be one of these market-clearing equations for each commodity, or altogether R equations. However, the equations (2) and (3) are all not independent, for if we are given the equations (2) and all but one of the equations (3), the last equation of the set (3) can be derived from the others; indeed, if we add together all of the equations (2) and then subtract from this total all but one of the equations (3), the remaining equation (3) is obtained as a residue. This is equivalent to saying that if each consumer balances his budget and if all but one of the markets are cleared, it is not possible for the last market not to clear. Or if all markets clear, and if all consumers but one balance their budgets, the last consumer must necessarily balance his budget: There is no one to whom he can be lending or from whom he can be borrowing. Thus of the N equations (2) and the R equations (3), one must be left out out as adding no new information, so that of these two types of equations together we have only $N + R - 1$ equations. Together with the $N(R - 1)$ equations of type (1), this makes $NR + R - 1$ equations.

From these equations we are to determine the amounts of each of the R commodities bought or sold by each of the N individuals (which together with the given amounts they had to start with, will tell us how much they consume of each commodity), and the prices. There are $R - 1$ and not R prices to be determined, for the price of whatever commodity is taken as numéraire is thereby fixed at 1; alternatively, we can consider that it is price ratios and not

the absolute prices that are important; indeed, if the numéraire is not itself a commodity, then the whole set of prices could be multiplied by any arbitrary factor without affecting the equilibrium. With the NR individual quantities to be determined, this makes $NR + R - 1$ quantities to be determined altogether by the $NR + R - 1$ equations.

The fact that we find a number of determining equations equal to the number of unknowns to be determined is an indication that the equilibrium sought is sufficiently but not excessively determined; that is, we have specified a sufficient number of conditions to determine a solution that in general will be unique, while on the other hand we do not have so many conditions to satisfy that it is not possible in general to find a solution that satisfies them all.

If the equations were less than the number of unknowns, this would mean that in general the information given is insufficient to determine the equilibrium position completely; there would generally be an infinite number of "solutions" that would satisfy the conditions laid down, and more information would be needed to determine which of the possible solutions would be the one actually arrived at in any particular instance. It is possible, in certain rare critical cases, for a unique solution to be specified by a number of equations less than the number of unknowns, but such a result is unlikely. For example, if we wish to find the coordinates of a point in space (three unknowns), specified as lying on the surfaces of both of two spheres (i.e., at specified distances from two points: two equations), all points on the circle formed by the intersection of the two spheres will be solutions; however, if the spheres happen to be tangent, the point of tangency will be the only solution. In this tangency case, the specified conditions will be very closely fulfilled even if the point moves about over a considerable range in the neighborhood of the point of tangency; one is led by this physical analogy to feel that in such rare cases where the solution happens to be fully determined by a number of conditions less than the number of unknowns, the solution may be somewhat less rigidly or exactly determined than where the number of constraints equals the number of degrees of freedom.

It is also possible for two or more of the conditions to be completely incompatible so that there is no solution at all, as when the two spheres in the above example are completely separate. In this case, there is no "real" solution (though in some cases a solution may be found if "imaginary" numbers involving $\sqrt{-1} = i$ are introduced and provided with some sort of interpretation).

On the other hand, if there are more equations than unknowns it will in general be impossible to satisfy all the conditions at once and one or more of them will have to be abandoned, unless of course the solution determined from a set of equations equal to the unknowns should happen to fulfill the remaining conditions. If this should happen, it would either be by sheer coincidence or because there existed an undetected relation between some of the equations whereby the fulfillment of a certain number of the conditions automatically required the fulfillment of some of the others. Thus if we happen to specify that a point in three-dimensional space shall be at specified distances from four other points, we shall in general encounter a contradiction. But it may happen that the point determined by three of the conditions does actually lie at the specified distance from the fourth point, and this may turn out to be so by reason of some property of the method by which the four specified distances were obtained. In the case of the general equilibrium equations, we saw, for instance, that one of the market-clearing equations was superfluous because it was already implied in some of the other equations of the system. If we had not observed this fact, we would have come out with more equations than unknowns and might have overhastily concluded that the solution was overdetermined and that one of the specified conditions would have to give way. Or if in addition we had failed to observe that our system would be equally well solved by any set of prices proportionate to those of any given solution, so that the general level of prices is indeterminate, we might have attempted to solve for R prices instead of $R - 1$ prices, and come out with an assertion that the conditions prescribe something that is not in fact determined by the conditions. Thus the operation of counting unknowns and equations is something to be done with considerable care.

Moreover, as we have seen, equality between the number of equations and the number of unknowns by no means guarantees a unique solution. Systems having several discrete solutions are not inherently unlikely, as was seen in FIGURE 35, for example. It is even possible, though here the possibility may be set down as inherently unlikely, for systems of equations to have an infinite number of solutions, along, say, a continuous segment of a curve, as in FIGURE 36, even though there is equality between the number of unknowns and the number of equations. It is also possible to produce systems with equations equal to unknowns that have no solutions, as in FIGURE 38, though here it is often possible to reinterpret matters so that a solution exists.

Another matter to consider in connection with a general equilibrium system is that the above system of equations was set up on the assumption that at the equilibrium point each trader would consume some positive amount of each commodity. Obviously, unless the commodities in the system are extremely broadly defined, for each trader there will in general be many commodities that are not consumed at all, in which case, as we saw on page 53, the equality between the marginal rate of substitution and the ratio of exchange may not hold, and the equations (1), sometimes called the Gossen equations, may have to be modified. To do this, it is convenient to introduce a new parameter for each individual, corresponding to the common ratios of (1), which may be called the "marginal utility of money" λ^a. Instead of the $R - 1$ equations (1),

for each individual, we will now have, for each of the R commodities, the following conditions to be met, for each individual:

$$\begin{aligned} &\text{either} \quad U_x^a = p_x\lambda^a \quad \text{and} \quad q_x^a > 0 \\ &\text{or} \quad U_x^a \leq p_x\lambda^a \quad \text{and} \quad q_x^a = 0 \end{aligned} \qquad (1')$$

We thus have always at least one equation, but in some cases it is the equality between the marginal utility of a dollar's worth of the commodity and the marginal utility of money, and in other cases it is the statement that none of the commodity is consumed. We thus have R equations for each trader, as compared with the $R - 1$ that we had previously, but of course we have introduced the new variable λ^a, so that the equality between equations and unknowns is undisturbed. We have, however, introduced a new type of condition, the inequality, but these are restrictions and not constraints; they may cut down the range within which the solution must lie, but they do not reduce the number of degrees of freedom in general. How to assure oneself that these inequality conditions can be met is actually a rather difficult problem, which cannot be fully dealt with here.

It should perhaps be further noted that in the above analysis the marginal utility of money is not the same thing as the marginal utility of the numéraire commodity as a commodity, and may differ from it if none of the numéraire commodity is actually consumed. The marginal utility of gold to a given individual, in terms of the possible direct uses he has for it, may be quite small relative to the marginal utility of the things he might purchase with the gold, if he had any left over.

The problem of determining whether a model has an equilibrium point and if so whether it is unique is thus not a simple one, and a complete treatment calls for rather high-powered mathematical analysis. For ordinary purposes, the matter may be summed up by saying that if the number of equations equals the number of unknowns, a certain presumption is created that the result is uniquely determined, while if the number of equations is greater than the number of unknowns, there is a certain presumption that no solution will satisfy all the conditions, and if the number of equations is less than the number of unknowns, there is a certain presumption that the solution is indeterminate. These, however, are mere presumptions, rebuttable through showing that the equations stand in certain special relations to one another. Accordingly, a comparison of the number of unknowns and the number of equations is useful chiefly as a preliminary check to see whether or not the number of conditions specified is sufficient, or whether more conditions must be sought, or whether some of them should be abandoned.

It is also important to remember that such a system of equations or relationships does not necessarily tell us whether or not the system will tend to approach the indicated solution or equilibrium if originally it starts from a situation in which the equilibrium conditions are not met, rather than engage in a series of movements that never reach an equilibrium. Nor does it even tell us, after an equilibrium has been previously reached, whether or not the system will tend to return to this equilibrium position if disturbed. To determine such matters we must add equations of motion that tell us how the system will react if it does not happen to be in equilibrium; a precise study of such behavior properly belongs to the study of dynamics. Nevertheless, if we are to make useful distinctions between equilibria that are stable and those that are unstable, it is appropriate, even as a part of a static analysis, to investigate the rudiments of the dynamics of the system, as was intimated on page 112.

Mathematical Appendix

TO MAXIMIZE a utility function $U(q_1, q_2, \ldots q_n)$ subject to the budgetary constraint

$$B = r - \sum_i p_i q_i = 0 \qquad 3.1$$

r being the limit on total expenditure and B the unspent margin, the more elegant and symmetric procedure is to form the function

$$V(q_1, q_2, \ldots q_n, \lambda) = U + \lambda B \qquad 3.2$$

and determine $\mathbf{q}^*$, λ^* in such a way that

$$V(\mathbf{q}, \lambda^*) \leq V(\mathbf{q}^*, \lambda^*) = V^* \qquad 3.3$$

and

$$V(\mathbf{q}^*, \lambda) \leq V^* \qquad 3.4$$

for all $\mathbf{q}$ and all λ. Clearly condition 3.4 requires $\partial V/\partial \lambda = 0$, i.e., $B = 0$, so that $V^* = U(\mathbf{q}^*) = U^*$. Then if there were a $\mathbf{q}^+$ for which $B(\mathbf{q}^+) = 0$, and $U(\mathbf{q}^+) > U^*$, we would have

$$V(\mathbf{q}^+, \lambda^*) = U(\mathbf{q}^+) + \lambda^* 0 > U^* = V^* \qquad 3.5$$

contradicting 3.3, so that if 3.3 and 3.4 are satisfied, the resulting $\mathbf{q}^*$ must be the maximizing set of quantities within the budgetary constraint.

Relation 3.3 then requires us to put

$$\frac{\partial V}{\partial q_i} = 0 \qquad 3.6$$

or $U_i - \lambda p_i = 0$. Together with $B = 0$, this gives $n + 1$ equations to be solved for the $n + 1$ unknowns $q_1, q_2, \ldots q_n, \lambda$. The equations 3.6 can be written $U_i/p_i = \lambda$; the left-hand side of this equation can be interpreted as the marginal utility of the amount of q_i that can be purchased for one unit of

money; λ can thus be considered the "marginal utility of money." The procedure can be thought of in terms of assuming that any budgetary surplus B can be spent at a price of unity on some extraneous commodity that has a constant marginal utility of λ (or if B is negative, the deficit must be made good by selling some good or service with a constant marginal disutility of λ). V is then the combined utility inclusive of that derived from this extraneous commodity, and in the maximization process λ is adjusted so that $B = 0$ and the dealings in this extraneous commodity vanish.

If the consumer is not required to spend all his income, the budgetary constraint becomes $B \geq 0$; since we must also exclude any negative marginal utility of money, in this case, we have $\lambda \geq 0$. 3.4 then requires either

$$B = 0 \quad \text{and} \quad \lambda \geq 0 \qquad 3.8$$

or

$$B > 0 \quad \text{and} \quad \lambda = 0 \qquad 3.9$$

If in addition we insist that for some of all of the q_i, negative values must be excluded, 3.3 then requires, instead of 3.6, the following alternatives:

either

$$\frac{\partial V}{\partial q_i} = U_i - \lambda p_i = 0 \quad \text{and} \quad q_i \geq 0 \qquad 3.10$$

or

$$\frac{\partial V}{\partial q_i} = U_i - p_i < 0 \quad \text{and} \quad q_i = 0 \qquad 3.11$$

for each i for which negative values of q_i are excluded, with 3.6 remaining in effect for the remaining values of i. Thus the marginal utility of a commodity is equal to the marginal utility of its money price when some is bought, but may be less than this when none is bought. If we have $\lambda = 0$, the point of satiation has been reached; if $B = 0$ satiation has just barely been reached, but if $B > 0$, this indicates that some of the available budget remains unused and there is money to burn. In this case we have $U_i = 0$ for all commodities i actually consumed, and $U_j \leq 0$ for all other commodities j. There is an obvious extension for the case where some of the q_i can take on only negative values, representing commodities sold rather than purchased, or services rendered.

The above conditions, while necessary to a maximum of utility, are not sufficient: They may be satisfied at a minimum or a saddle point: for example, the indifference curves might be concave to the origin at the point in question. For two commodities, it is sufficient that we satisfy the convexity conditions of 2.22; this can be written in determinant form:

$$\begin{vmatrix} 0 & U_x & U_y \\ U_x & -U_{xx} & -U_{xy} \\ U_y & -U_{xy} & -U_{yy} \end{vmatrix} < 0$$

It is sufficient if this is negative; it is necessary that it not be positive. If it is zero, we have a special case in which further and rather difficult investigation is necessary if we are to determine whether we have a true maximum. Fortunately, this case is unlikely to arise in practice; it represents the case where there is a high order of contact between the budget line and the indifference curves.

The algebra involved in deriving the conditions for more than two commodities is fairly involved, but the results can be summarized as follows.

Let T be the set of commodity dimensions i for which q_i is positive at the point of equilibrium under investigation, and let S be any subset of T. (Actually, T may be extended to include those dimensions j for which by coincidence the marginal utility equals the marginal utility of its price, even though $q_j = 0$.) Define

$$D_S = \begin{vmatrix} 0 & \mathbf{U}_S' \\ \mathbf{U}_S & -\mathbf{U}_{SS} \end{vmatrix} \qquad 3.10$$

where $\mathbf{U}_S$ is a column vector of the marginal utilities of the commodities in S, $\mathbf{U}_S'$ is the corresponding row vector, and $\mathbf{U}_{SS}$ is the matrix of the second derivatives of U with respect to the commodities in S. Then it is necessary, for a maximum of U to exist subject to the budget constraint, that D_S never be positive for any set $S \subseteq T$. It is sufficient for the existence of at least a local maximum if the first order conditions, $U_i = \lambda p_i$, are satisfied and in addition D_S is negative for each of some sequence of sets S, starting with some two-element set and adding one commodity at a time until T is reached. If the necessary conditions are met but none of the sets of sufficient conditions are met, which will occur when one of the $D_S = 0$, the question of whether a true maximum exists locally at the point in question can only be resolved by more involved and quite difficult investigations, usually involving third- and possibly higher-order derivatives. If S has only one element, then

$$D_S = \begin{vmatrix} 0 & U_1 \\ U_1 & -U_{11} \end{vmatrix} = -U_1^2$$

and the test is met trivially.

Engel curves. In what follows discussion will be limited to the set of commodities T for which $U_i = \lambda p_i$ holds at the equilibrium point, including chiefly those commodities where q_i is positive at the equilibrium point. q and λ constitute $n + 1$ unknowns to be determined by the $n + 1$ equations

$$U_i = \lambda p_i \qquad (i = 1, 2, \ldots n) \qquad 3.5$$

$$\sum_i p_i q_i = r \qquad 3.1$$

as functions of the $n + 1$ parameters $\mathbf{p}$ and r. It is natural to think of these equations as being solved so as to give $\mathbf{q}$ and λ explicitly as function of $\mathbf{p}$ and

r; if we then hold $\mathbf{p}$ constant and vary r, $\mathbf{q}$ will trace out an Engel curve. For small changes in r, the "income effect" is then $(\partial \mathbf{q}/\partial r)_p$, the p outside the parenthesis being added to make explicit which are the variables that are being held constant during the partial differentiation. In order to investigate this effect, we carry out the partial differentiation of 3.1 and 3.5 with respect to r, keeping $\mathbf{p}$ constant, we get, from 3.1

$$\sum_j p_j \left(\frac{\partial q_j}{\partial r}\right) = 1 \qquad 3.14a$$

and from 3.5,

$$p_i \left(\frac{\partial \lambda}{\partial r}\right) - \sum_j U_{ij} \left(\frac{\partial q_j}{\partial r}\right) = 0 \qquad (i = 1, 2, \ldots n) \qquad 3.14b$$

Substituting for p_i and p_j from 3.5, we get

$$\sum_j U_j \left(\frac{\partial q_j}{\partial r}\right) = \lambda \qquad 3.15a$$

and

$$U_i \left(\frac{1}{\lambda}\right) \left(\frac{\partial \lambda}{\partial r}\right) - \sum_j U_{ij} \left(\frac{\partial q_j}{\partial r}\right) = 0 \qquad (i = 1, 2, \ldots n) \qquad 3.15b$$

Regarding the variables $(1/\lambda)$, $(\partial\lambda/\partial r)$, and $(\partial q_j/\partial r)$ as the $n + 1$ unknowns to be determined by these equations, we note that the determinant of the coefficients of these variables on the left side of these equations is precisely the D_S of 3.10 above, with $S = T$. Put $D_{T;i,j}$ for the cofactor of the ith row and jth column in the expansion of the determinant D_T. Then, using Cramer's rule for the solution of these equations, we have

$$\frac{1}{\lambda}\frac{\partial \lambda}{\partial r} = \frac{D_{T;0,0}}{D_T}\lambda \quad \text{and} \quad \frac{\partial q_j}{\partial r} = \frac{D_{T;0,j}}{D_T}\lambda \qquad 3.16$$

The only thing that can be said a priori about the signs of these effects is that if the p_j are all positive, not all of the $(\partial q_j/\partial r)$ can be negative, which indeed follows from 3.14a. 3.16 will determine whether any particular good is "inferior" at a given point.

Offer curves. To examine the effect of a variation in a given price, with income and all other prices constant (this is the "Walrasian" type of demand curve, as distinguished from what is sometimes termed the "Marshallian" type of demand curve) we differentiate 3.1 and 3.5 with respect to p_k, and after making the substitutions for p_i from 3.5 as in the previous case, the results become

$$\sum_j U_j \left(\frac{\partial q_j}{\partial p_k}\right) = -\lambda q_k \qquad 3.17a$$

$$U_i \frac{1}{\lambda}\frac{\partial \lambda}{\partial p_k} - \sum_j U_{ij}\frac{\partial q_j}{\partial p_k} = -\lambda \delta_{ik} \qquad (i = 1, 2, \ldots n) \qquad 3.17b$$

where δ_{ik} is the Kronecker δ, defined to have the value 1 for $i = k$ and 0 otherwise. Solving again by Cramer's rule, we have

$$\frac{1}{\lambda}\frac{\partial \lambda}{\partial p_k} = \frac{1}{D_T}[-\lambda q_k D_{T;0,0} - \lambda D_{T;k,0}] \qquad 3.18a$$

$$\frac{\partial q_j}{\partial p_k} = \frac{1}{D_T}[-\lambda q_k D_{T;0,j} - \lambda D_{T;k,j}] \qquad 3.18b$$

Comparing 3.18b with 3.16b, we see that 3.18b can be written

$$\frac{\partial q_j}{\partial p_k} = -q_k\frac{\partial q_j}{\partial r} - \lambda\frac{D_{T;k,j}}{D_T} \qquad 3.19$$

The first term on the right can thus be called the "income effect" and the second term the "substitution effect." The direction of the income effect is obviously opposite to that of the price change in p_k if q_k is positive, whenever j is a normal good, which by definition means that $(\partial q_j/\partial r)$ is positive, and in the same direction when j is an inferior good; these comments are, however, reversible if q_k is negative, which might be the case, for example, if commodity k is one being sold rather than purchased by the individual in question.

The last term of 3.19 can be called the "substitution effect," for which we can use the symbol

$$S_{kj} = -\lambda\left(\frac{D_{T;k,j}}{D_T}\right) \qquad 3.20$$

Since D_T is a symmetric determinant,

$$D_{T;k,j} = D_{T;j,k} \qquad 3.21$$

so that $S_{kj} = S_{jk}$. The substitution effect is thus symmetrical: the substitution effect of a drop in the price of a commodity x on the consumption of y is equal to the effect of a drop in the price of commodity y on the consumption of x, both effects having the dimension

$$\frac{(\text{units of } x)\ (\text{units of } y)}{(\text{units of numéraire})}$$

The income effect, however, is not symmetrical; indeed, we may have $(\partial q_j/\partial r)$ positive while $(\partial q_k/\partial r)$ is negative. In general, therefore, $(\partial q_j/\partial p_k) \neq (\partial q_k/\partial p_j)$.

If $j = k$, so that q_j and p_k are referring to the same commodity, we have $D_{T;k,j} = D_{T;j,j} = D_V$ where $V = T - (j)$; from 3.11 we know that neither D_T nor D_V can be positive if utility is being maximized, hence S_{jj} cannot be positive; i.e., the substitution effect of an increase in a price on its own commodity is never positive.

More generally, since replacing the jth column of D_T by a duplicate of its first column gives a zero determinant, we have $\sum_k U_k D_{T;k,j} = 0$, so that $\sum_k U_k S_{kj} = 0$. Since the U_k are all non-negative, this means that at least some of the S_{kj} must be sufficiently positive to offset the negative term S_{jj}. If then we define complementarity between two distinct commodities as meaning that S_{kj} is negative—i.e., increasing the price of one, income effects aside, tends to decrease the consumption of the other, and, conversely, that substitution between two commodities means S_{kj} is positive—then complementarity between distinct commodities must on the whole be outweighed by substitutability, at least when the marginal utilities are used as weights. In particular, if there are only two commodities, S_{12} cannot be negative: a rise in the price of one of the two commodities cannot produce a decrease in the consumption of the other, except via the income effect.

The effects of price and income changes on utility can be obtained by the summation of the effects mediated by the various changes in quantities:

$$\frac{\partial U}{\partial r} = \sum_j U_j\frac{\partial q_j}{\partial r} = \lambda \qquad \text{(from 3.15a)} \qquad 3.24a$$

and

$$\frac{\partial U}{\partial p_k} = \sum_j U_j\frac{\partial q_j}{\partial p_k} = -\lambda q_k \qquad \text{(from 3.17a)} \qquad 3.24b$$

3.24a can be considered further justification for calling λ the marginal utility of money or, perhaps better, income.

It is instructive to consider a simultaneous variation of p_k and r in such a way as to keep U constant, all prices other than p_k being kept constant. To avoid ambiguity, we adopt a notation in which the variables being kept constant during a differentiation are indicated by subscripts outside parentheses enclosing the derivative. We then have

$$\left(\frac{\partial U}{\partial p_k}\right)_{p_j} = \left(\frac{\partial U}{\partial p_k}\right)_{p_j,r} + \frac{\partial U}{\partial r}\left(\frac{\partial r}{\partial p_k}\right)_{U,p_j} = 0, \qquad 3.25$$

which becomes, using 3.24:

$$-\lambda q_k + \lambda\left(\frac{\partial r}{\partial p_k}\right)_{U,p_j} = 0, \qquad \text{whence} \left(\frac{\partial r}{\partial p_k}\right)_{U,p_j} = q_k \qquad 3.26$$

Another way of looking at this question is to think of all the variables U, λ, r, $\mathbf{p}$, and $\mathbf{q}$ as varying simultaneously in a unique way as functions of some, parameter t, not necessarily identified with "time" in a calendar sense. Considering the relation between U and $(r, \mathbf{p})$ as given by 3.1 and 3.5 we have

$$\frac{dU}{dt} = \frac{\partial U}{\partial r}\frac{dr}{dt} + \sum_i \frac{\partial U}{\partial p_i}\frac{dp_i}{dt} = \lambda\frac{dr}{dt} - \lambda\sum_i q_i\frac{dp_i}{dt} \qquad 3.27$$

If we now require that the variation in the variables be such that $(dU/dt) = 0$, and $(dp_i/dt) = 0$ for all $i \neq k$, we have

$$0 = \lambda \frac{dr}{dt} - \lambda q_k \frac{dp_k}{dt}, \quad \text{or} \quad \frac{\left(\frac{dr}{dt}\right)}{\left(\frac{dp_k}{dt}\right)} = q_k \qquad 3.28$$

The effects of such a combined variation of p_k and r on the quantities q_j is then given by

$$\left(\frac{\partial q_j}{\partial p_k}\right)_{U, p_{i\neq k}} = \left(\frac{\partial q_j}{\partial p_k}\right)_{r, p_{i\neq k}} + \left(\frac{\partial q_j}{\partial r}\right)_p \left(\frac{\partial r}{\partial p_k}\right)_{U, p_{i\neq k}} \qquad 3.29$$

$$= -\lambda \frac{q_k D_{T:0,j} + D_{T:k,j}}{D_T} + \lambda \frac{D_{T:0,j}}{D_T} q_k \qquad 3.30$$

$$= -\lambda \frac{D_{T:k,j}}{D_T} = S_{kj} \qquad 3.31$$

where 3.30 is obtained by putting 3.16b and 3.18b in 3.29. The substitution term S_{kj} is thus simply the net effect on q_j of a change in p_k combined with a "compensating" change in r just sufficient to permit the consumer to maintain the same level of U.

In the same way, for the effect on the marginal utility of money of a compensated price change we have

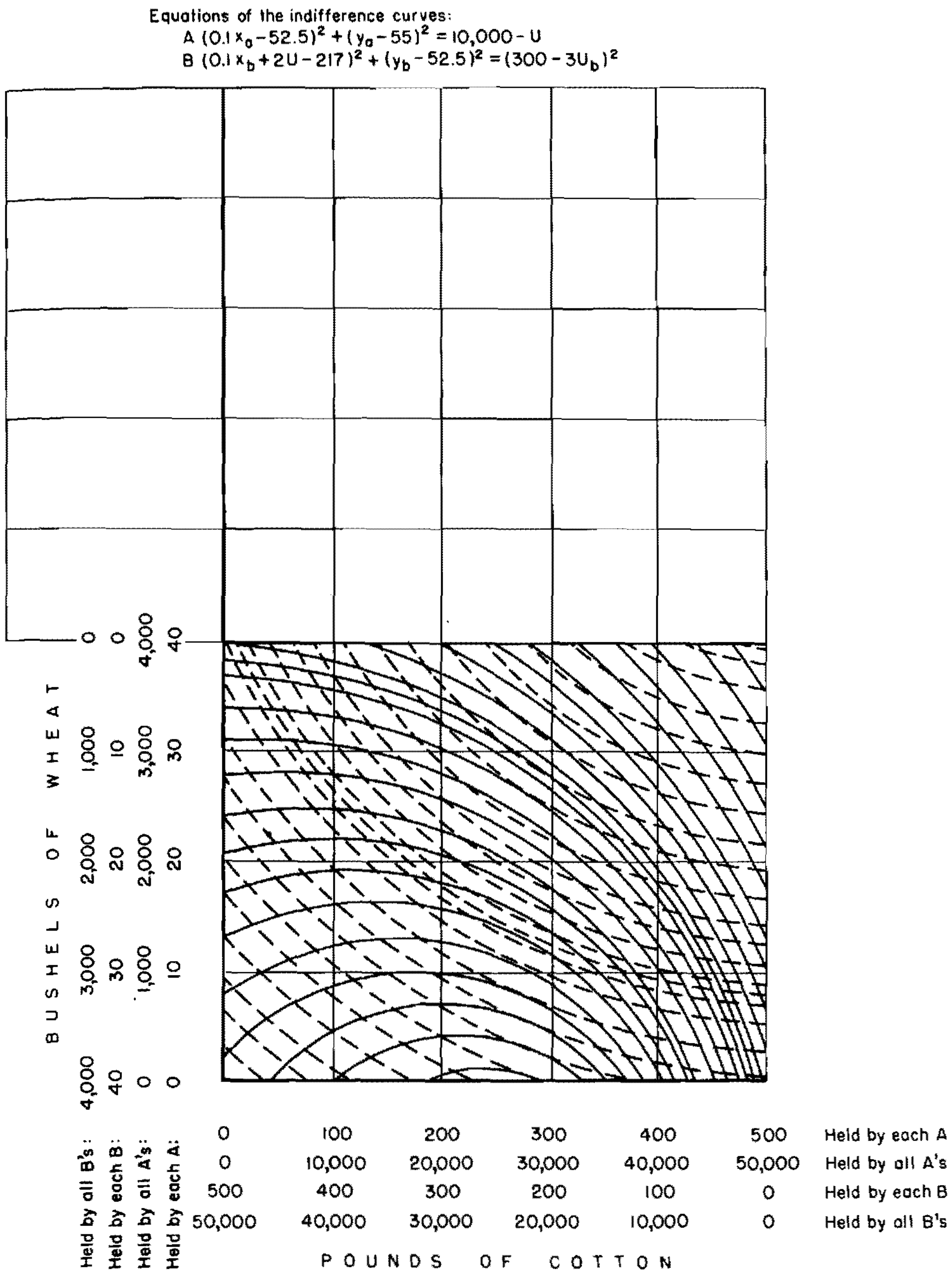
$$\frac{1}{\lambda}\left(\frac{\partial \lambda}{\partial p_k}\right)_{U, p_{i\neq k}} = -\frac{\partial q_k}{\partial r} \qquad 3.32$$

making use of 3.16a and 3.18a; the effect is thus opposite in sign to the income effect, and is zero when the income effect is zero.

EXERCISES

I

IN THE graph reproduced on page 131, any point in the lower-right-hand rectangle represents the division of the total supply of two commodities (say 4,000 bushels of wheat, y, and 50,000 pounds of cotton, x) between two groups A and B of 100 individual consumers each. The dashed curves represent the indifference curves of the A's, assuming that all the A's have the same tastes and that the amounts of the various commodities in the possession of the A group is equally distributed among the members of the A group. (The curves may be considered the individual indifference maps of the individual A's if the scales are divided by the number of A's in the group.) Similarly, the solid curves represent the indifference curves of B.



1. Draw the contract curve (the locus of points such that there is no further exchange that is desirable simultaneously to both A's and B's). Mark the curve *UUU*.

2. Suppose that initially the 100 A's hold the entire supply of commodity y (equally divided among them), say 40 bushels of wheat each, while 100 B's hold similarly the entire supply of x, say 500 pounds of cotton each.

a. Locate the initial point and mark it S.

b. Designate by *MMMMM* the boundary of the area of mutual benefit dependent on S, within which both the A's and the B's will be better off than at S.

c. Draw the offer curves of the A's and the B's (these can be considered either individual or aggregate curves, depending on the scale to which they are read).

d. Draw the demand curve of the A's and the supply curve of the B's. (These may be conveniently drawn in the space left at the top right of the page, using the unit price laid off at the top left.)

e. If both the A's and the B's behave competitively, what will be the results? Mark the point C, and enter the results in the table at the end of these instructions.

f. If the B's organize a simple monopoly market and set a price at which the A's may buy as much as they please and the A's act competitively, what will be the results if the B's succeed in selecting the price most favorable to them? Mark the point M_b and enter the results in the table.

g. If the B's organize a monopoly and are able to specify both the price and the quantity that the A's are to be permitted to buy (and the A's cannot resell among themselves), what would be the result that would be approached if the B's succeed in exploiting their position to the fullest possible extent? (It is assumed, of course, that the A's can always refuse an offer that makes them worse off than they were initially.) Mark the point D_b and enter the result in the table.

h. If the A's organize a simple monopsony and set the price but not the quantity, and the B's behave competitively, mark the point M_a, and enter the result in the table.

i. If the A's organize a discriminating monopsony and set both the price and the quantity they will agree to purchase, which transaction the B's may only accept or reject individually without resale among themselves, enter the result which the A's will try to approach in the table. Mark the point D_a.

3. Suppose that initially both cotton and wheat were divided equally among A's and B's.

a. Designate the starting point E.

b. Indicate by *mmm* the area of mutually advantageous trades that might be made from E as a starting point.

c. Under conditions of competition, what would be the result of starting from E? Enter the results in the table and designate the point F.

d. What is the locus of starting points that would yield the point F under conditions of competitive trading? Designate the locus GG.

e. From the starting point of # 2, how much wheat would have to be taken from the A's and given to the B's (before they come to the market, for example, by an income tax), to provide that the subsequent competitive trading would end at F? ________ bushels each.

4. Compare (a) the income tax with (b) collective bargaining as methods of equalizing the distribution of income.

	Price of cotton (in bushels of wheat per pound of cotton)	Amounts traded		Final share of each A		Final share of each B	
		Wheat, bushels	*Cotton, pounds*	*Wheat, bushels*	*Cotton, pounds*	*Wheat, bushels*	*Cotton, pounds*
2e. Competition							
f. Simple monopoly by B							
g. Discriminatory monopoly by B							
h. Simple monopsony by A							
i. Discriminatory monopsony by A							
3c. Competition (from initial equal distribution)							

II

A POINT in the rectangle at the bottom of the graph (p. 134) represents the division of the total supply of 6,000 yards of fabric and 70,000 feet of lumber between 100 A's and 100 B's. The solid curves represent the indifference curves of the A's, who are assumed to share their portion of the commodities equally among themselves and to have identical tastes; similarly, the dashed curves represent the indifference curves of the B's.

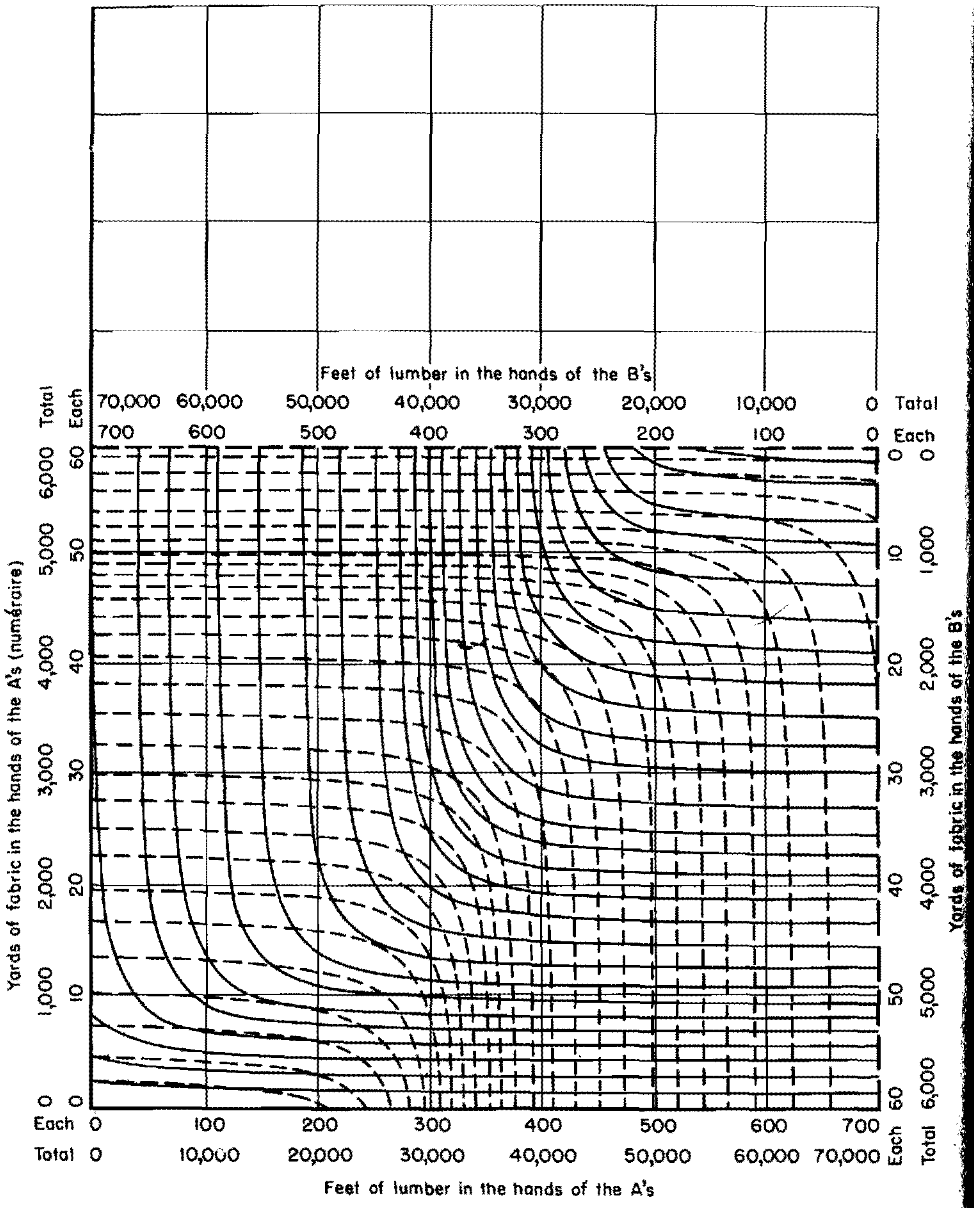
1. Observe the indifference curves from both directions. What is the relation between the tastes of the A's and the B's?

2. Initially, the A's have all the fabric and the B's all the lumber.

a. Draw the offer curves of A and B.

b. Draw A's demand curve and B's supply curve in the space above the indifference curves. (Note: For lack of space a unit scale for prices was not provided; the price can, however, be transferred readily with a pair of dividers using a convenient arbitrary distance as the unit and measuring for each price line the vertical distance corresponding to this unit horizontal distance.)

Equation of the curves: (unit = 0.7") = $(x - 30 + \frac{2}{u} - 3u)(y - 15 + \frac{6}{u} - u) = 18$



c. Under competitive conditions, what are the points of equilibrium? Which of these are points of stable equilibrium? Mark them C_s. Which are points of unstable equilibrium? Mark them C_u.

d. Draw the contract curve. How is this related to the points of equilibrium?

3. Initially, both the fabric and the lumber are equally divided between the *A*'s and the *B*'s.

a. Locate the starting point. Mark it *E*.

b. What will happen when the *A*'s and the *B*'s come to market?

c. What are the necessary conditions for economic trade?

(Refer also to Exercise I, 3c.)

GENERAL NOTE: The indifference curves may be conceived to refer to a case where fabric and lumber are to be used by the consumer for the manufacture of furniture for his own use, and that the proportion of these materials used in a given type of furniture is fairly fixed (hence the fairly marked complementarity shown by the curves). As income increases (or, more properly, as the consumption of furniture increases), consumers insist on styles of furniture requiring more fabric and relatively less wood, with the result that the income-consumption curves bend upward.